A comparison of global sensitivity analysis used in water systems

Arinala Randrianantoandro, Marc Neumann, François Anctil & Peter Vanrolleghem

2012 Statistics and Control Meeting

McMaster University, Hamilton, Ontario

24 / 05 / 2012

UNIVERSITÉ LAVAL

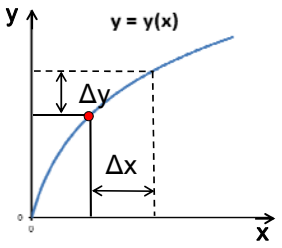
modelEAU

Outline

- Sensitivity Analysis (SA)
- Global Sensitivity Analysis (GSA)
  - Morris Screening
  - Standardized Regression Coefficient (SRC)
  - Extended Fourier Amplitude Sensitivity Testing (eFAST)
- GSA Comparison
- Conclusion

Sensitivity Analysis - SA

- **What is SA?**
  - How is the model output affected by the variation of model factors (parameters+inputs)?
- Sensitivity Index: $Is_i = \Delta y/\Delta x$
  - $Is_i >>$: x is important
  - $Is_i <<$: x is non-influential



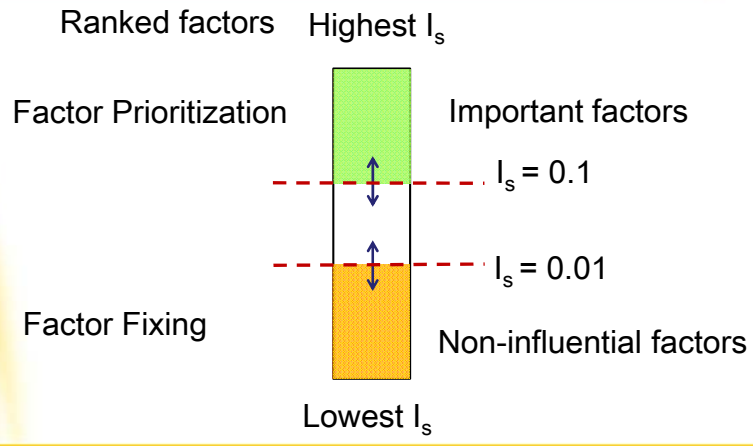
Sensitivity Analysis – Uses of SA

- Model diagnosis
  - How is the model output affected by changes in the factor values?
- Identifiability analysis
  - Which subsets of factors are identifiable?
- Uncertainty analysis
  - How much does each (uncertain) factor contribute to the uncertainty in the model output

Sensitivity Analysis – Objectives

- Factors prioritization
  - Identifying which factors have the greatest contribution to model output uncertainty
- Factors fixing
  - Identifying which factors are non-influential

Sensitivity Analysis – Objectives

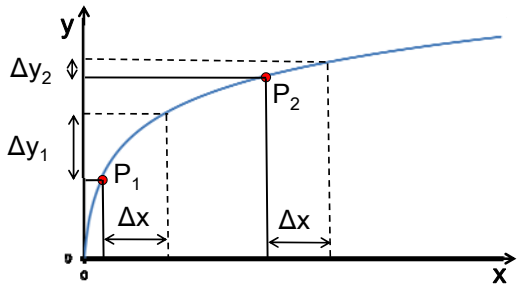


Outline

- Sensitivity Analysis (SA)
- Global Sensitivity Analysis (GSA)
  - Morris Screening
  - Standardized Regression Coefficient (SRC)
  - Extended Fourier Amplitude Sensitivity Testing (eFAST)
- GSA Comparison
- Conclusion

Global Sensitivity Analysis - GSA

- **Why GSA? not LSA** (Local SA)
- To have an overall idea of the sensitivity



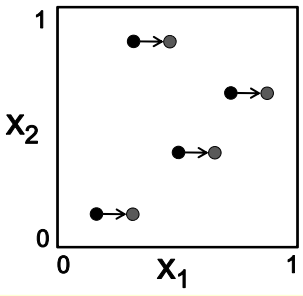
Global Sensitivity Analysis - GSA

- **3 examples** of GSA

GSA	Factors Prioritization	Factors Fixing
Morris		X
SRC	X	
eFAST	X	X

Global Sensitivity Analysis – How?

- Choose the output variable: y
  - extreme value, average value, MSE, ...
- Define the factors space: {x}
- Sample x_i
- Run model
- Repeat simulation
- Replicate



GR4J model

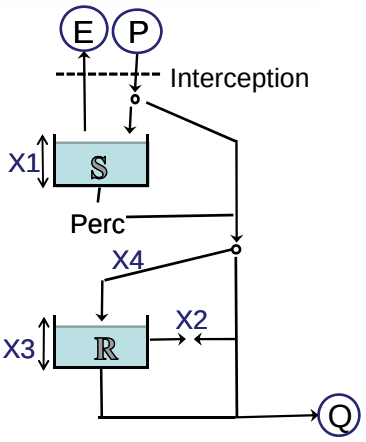
"rainfall-runoff" model
(Perrin *et al*, 2003)

- Inputs:

P: Precipitation (rain)
E: Evapotranspiration

- Output:

Q: River Flow



Global Sensitivity Analysis - Morris

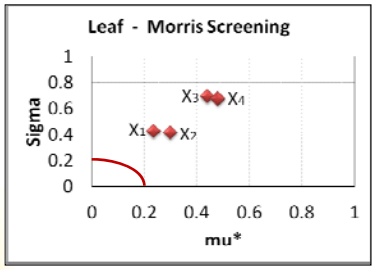
- Screening based method
- Sampling: one factor at a time
- Distribution of "elementary effects"

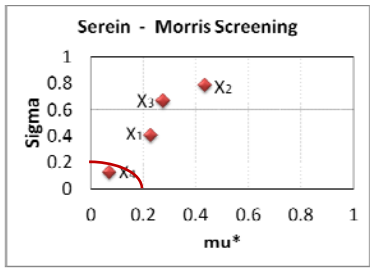
$$d_i(x_i) = \frac{y(x_1,\ldots,x_{i-1},x_i+\Delta,x_{i+1},\ldots,x_k) - y(x_1,\ldots,x_{i-1},x_i,x_{i+1},\ldots,x_k)}{\Delta}$$

- Ranking of factors on two scales:
  - Importance: $mu^*=mean(|d_i|)$
  - Non-linearity and/or interactions: $sigma=sd(d_i)$

Global Sensitivity Analysis - Morris

- Example: GR4J model – 2 catchments





- X_4 (Serein) non-influential: can be fixed

Global Sensitivity Analysis - SRC

- Regression based method
- Linear regression on output simulations

$$y = \sum b_i . x_i + a$$

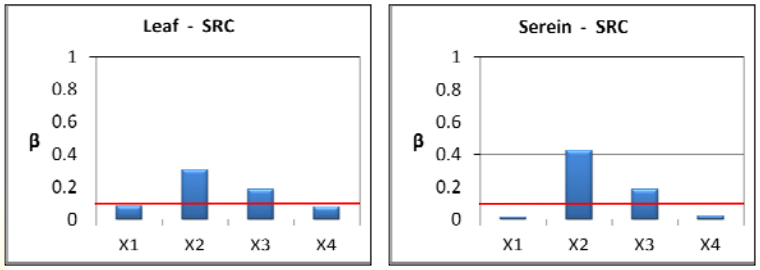
$$\beta_i = b_i . \frac{\sigma_{x_i}}{\sigma_y}$$

Standardised Regression Coefficients (SRC)

- Analysis valid if $R^2 > 0.7$ (Saltelli *et al.*, 2004) (Quasi-linear model)

Global Sensitivity Analysis - SRC

- Regression method: **SRC** (Factors **prioritization**)



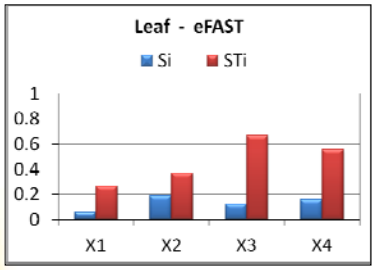
- X_2 and X_3 are important: to be estimated

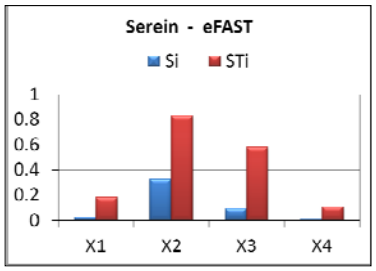
Global Sensitivity Analysis - eFAST

- Variance based method
- First order effect of x_i: S_i
  - gives the expected variance reduction if x_i was known perfectly (no variation)
- Total effect of x_i: S_{Ti}
  - gives the expected fraction of the total variance that would be left if ($\{x_{1...k}\} \setminus \{x_i\}$) were fixed
- Interaction: $S_{Ti} - S_i$

Global Sensitivity Analysis - eFAST

- **eFAST:** Factors **prioritization** (S_i) & **fixing** (S_{Ti})





- X_1 (Leaf), X_1,X_4(Serein): not important (blue bars)
- They cannot be fixed (red bars)

Global Sensitivity Analysis

- Number of required simulation runs to reach convergence (of parameter groups)

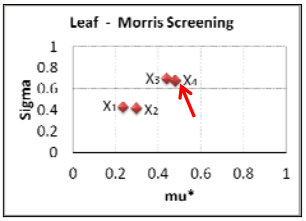
	N° factors	GSA		
		Morris	SRC	eFAST
Suggested	**k**	**[4-8]*(k+1)**	**[500-1000]**	**[500-1000]*k**
GR4J	4	50*(k+1)	1000	1000*k
MORDOR10	10	100*(k+1)	5000	2000*k

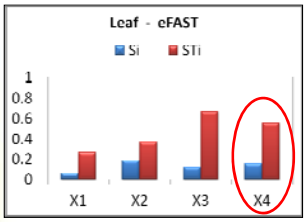
Outline

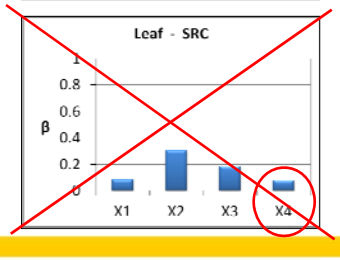
- Sensitivity Analysis (SA)
- Global Sensitivity Analysis (GSA)
  - Morris Screening
  - Standardized Regression Coefficient (SRC)
  - Extended Fourier Amplitude Sensitivity Testing (eFAST)
- GSA Comparison
- Conclusion

GSA Comparison – GR4J

- SRC
  $R^2 = 0.14 < 0.7$







MORDOR10 model

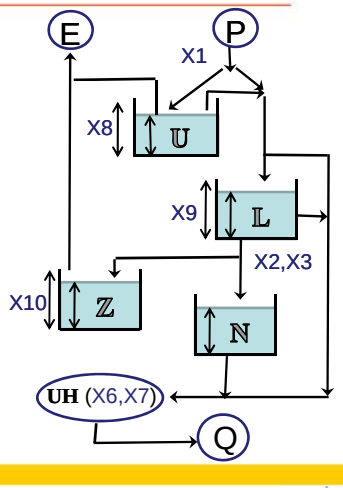
"rainfall-runoff" model
(Paquet, 2004)

- Inputs:

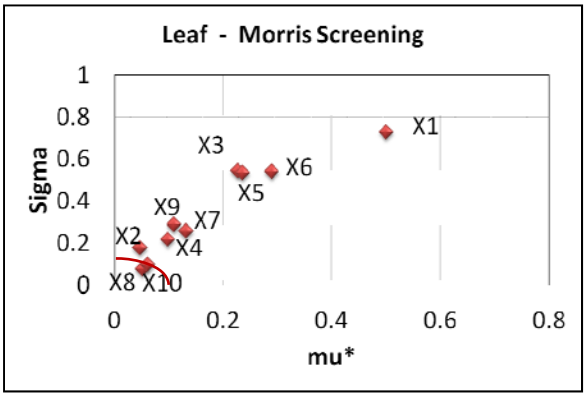
P: Precipitation (rain)
E: Evapotranspiration

- Output:

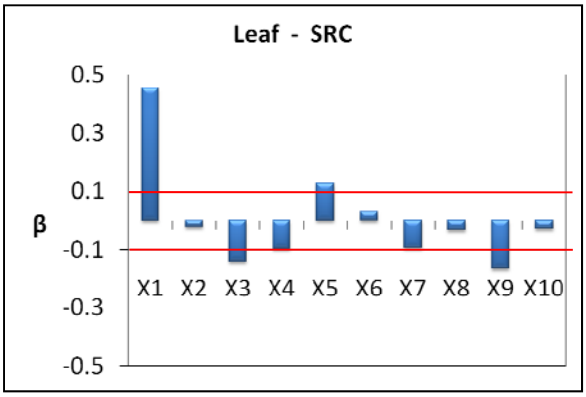
Q: River Flow



Morris – MORDOR10



SRC – MORDOR10

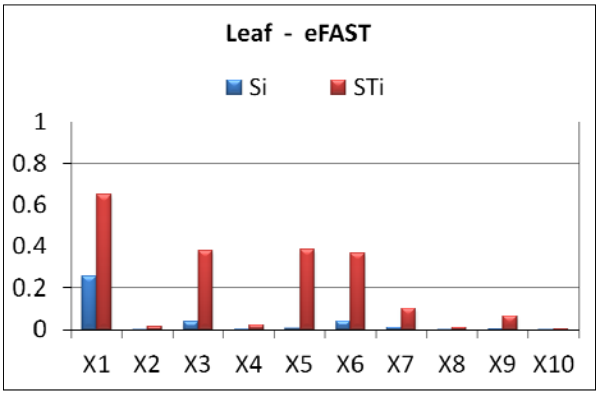


UNIVERSITÉ LAVAL

23

modelEAU

eFAST – MORDOR10

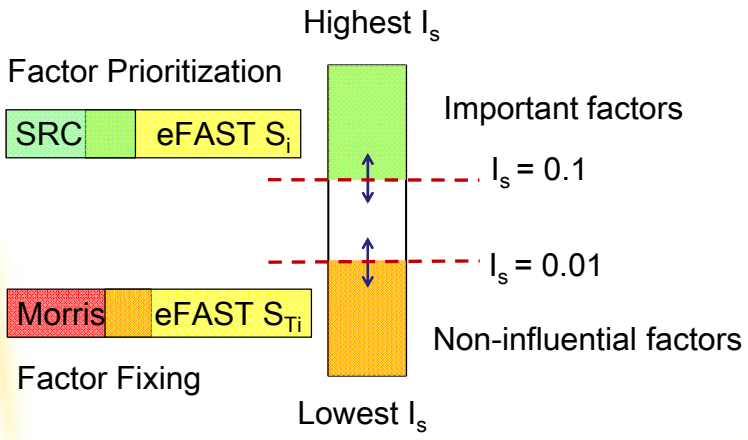


UNIVERSITÉ LAVAL

24

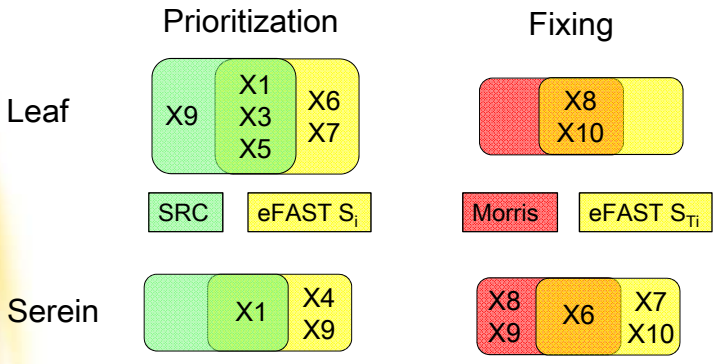
modelEAU

GSA Comparison - MORDOR10



GSA Comparison – MORDOR10

- **Hydrology: 10 factors**



Outline

- Sensitivity Analysis (SA)
- Global Sensitivity Analysis (GSA)
  - Morris Screening
  - Standardized Regression Coefficient (SRC)
  - Extended Fourier Amplitude Sensitivity Testing (eFAST)
- GSA Comparison
- Conclusion

Conclusion

- GSA: how output is affected by model factors?
- SRC and eFAST (S_i) are used for factors prioritization: most important factors
- Morris and eFAST (S_{Ti}) are used for factor fixing: non-influential factors

Conclusion

- GSA results are case study dependent
- GSA methods don't always give the same grouping of factors
- A number of method settings needs study:
  - Number of simulations needed (convergence)
  - Cut-off values for grouping