



The Bürger-Diehl model: The right integrator for the right model



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Introduction

The Bürger-Diehl settling model is a physically-sound spatially discretized settling model. We show that its complexity does not prevent acceptable computation times at various spatial discretization

Objectives

- Compare the results obtained with the Bürger-Diehl model under different spatial discretization
- Propose optimal solver settings to solve the model

Influence of the spatial discretization

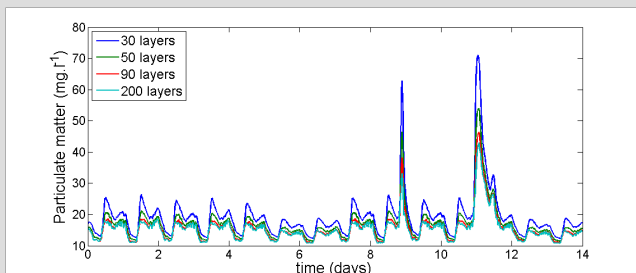


Figure 1: Concentration of particulate matter exiting the secondary clarifier with respect to the spatial discretization of the Bürger-Diehl model. As the number of layer increases, the simulation converges to a stable solution.

Steady state simulations

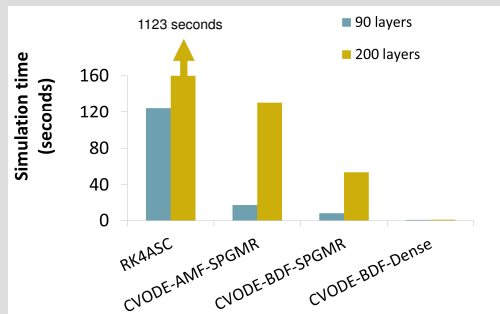
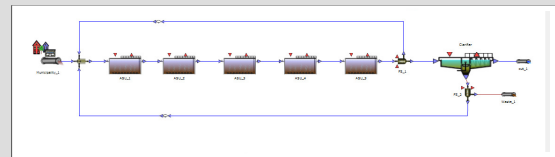


Figure 2: Simulation time to reach steady state. Using exact Newton methods provided impressive acceleration of the computation with the CVODE-BDF-Dense algorithm.

Methodology

- The Bürger-Diehl model was applied to the **Benchmark Simulation Model 1**. Two simulations were performed:
 - 100 days with constant inputs to steady state
 - 14 days of dynamic inputs with two storm events



- Four spatial discretization were tested in the secondary settler: 30, 50, 90 and 200 layers
- Four solver configurations were compared:
 - Runge-Kutta 4 with adaptive time step (**RK4ASC**)
 - CVODE from the SUNDIALS library using Adams-Moulton formula and an internal iterative solver (SPGMR) iterative solver (**CVODE-AMF-SPGMR**).
 - CVODE using the Backward Differential Formulas and the SOGMR iterative solver (**CVODE-BDF-SPGMR**).
 - CVODE using the Backward Differential Formulas and the Newton-Raphson solver operating on dense matrices (**CVODE-BDF-Dense**).

Dynamic simulations

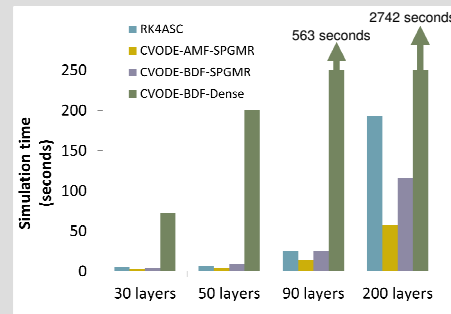


Figure 3: Simulation time with dynamic inputs. The CVODE-AMF-SPGMR provided the overall optimal computation time. The CVODE-BDF-Dense was always the worst solver under dynamics.

Key findings

- The spatial discretization has an important effect on accuracy and simulation time.
- The steady state simulation was fastest when using the CVODE-BDF-Dense solver.
- Dynamic simulation was fastest with the stiff solver CVODE-AM-SPGMR.