

**Development and numerical solution
of a multiclass model
for settling processes in
water resource recovery facilities**

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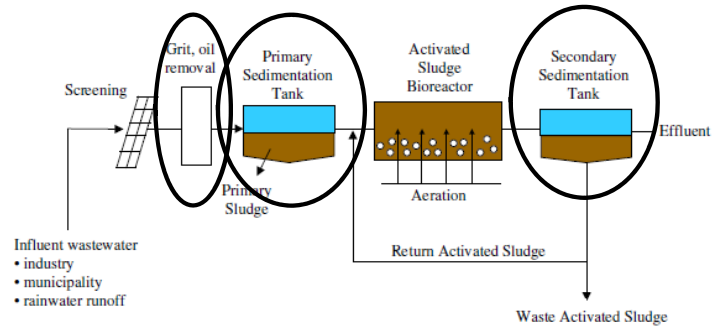
BIOMATH



Outline

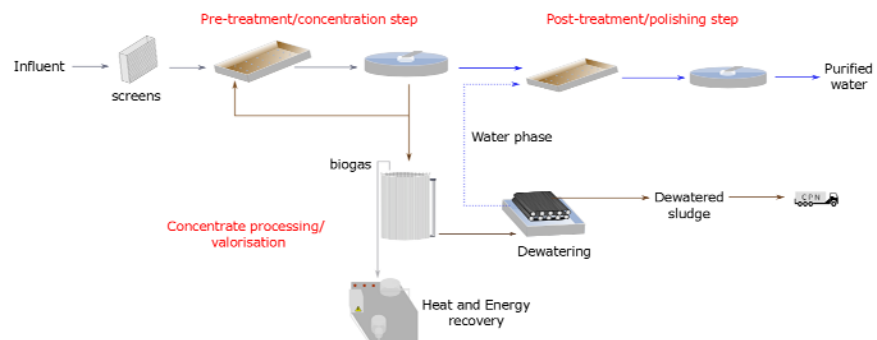
- Settling in wastewater treatment
- Variability in settling behaviour
- Available models for settling processes
- Unified modelling framework
- Numerical solution
- Simulation examples
- Conclusions

Settling processes



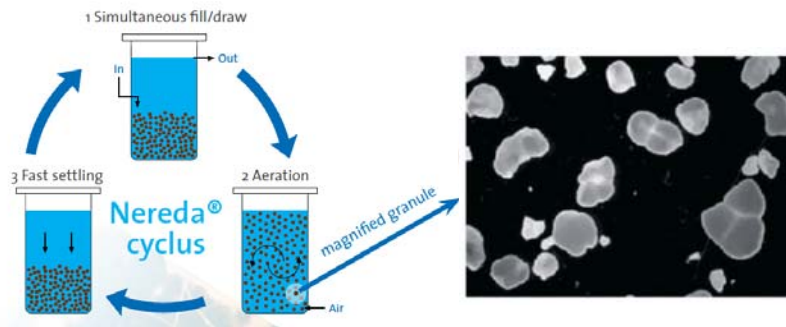
Settling processes

- Emerging technologies: A/B system

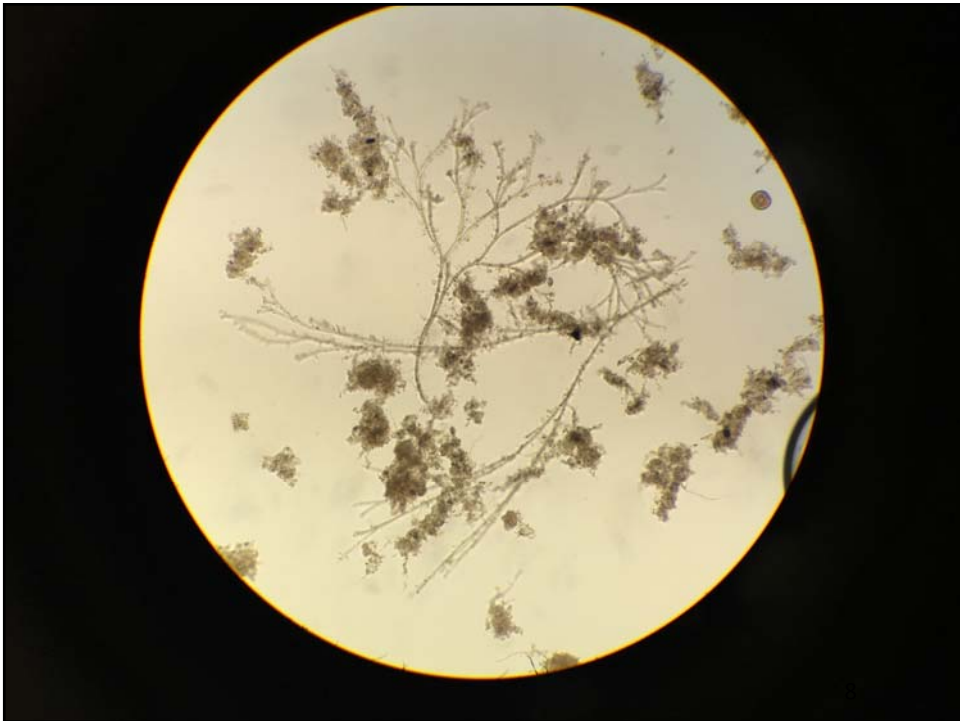


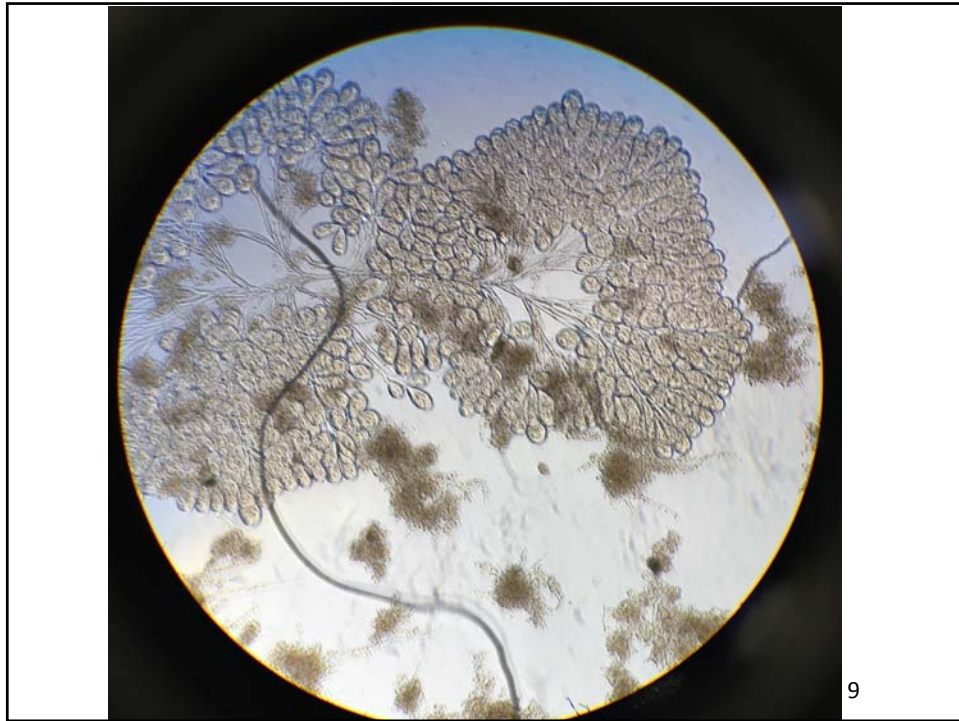
Settling processes

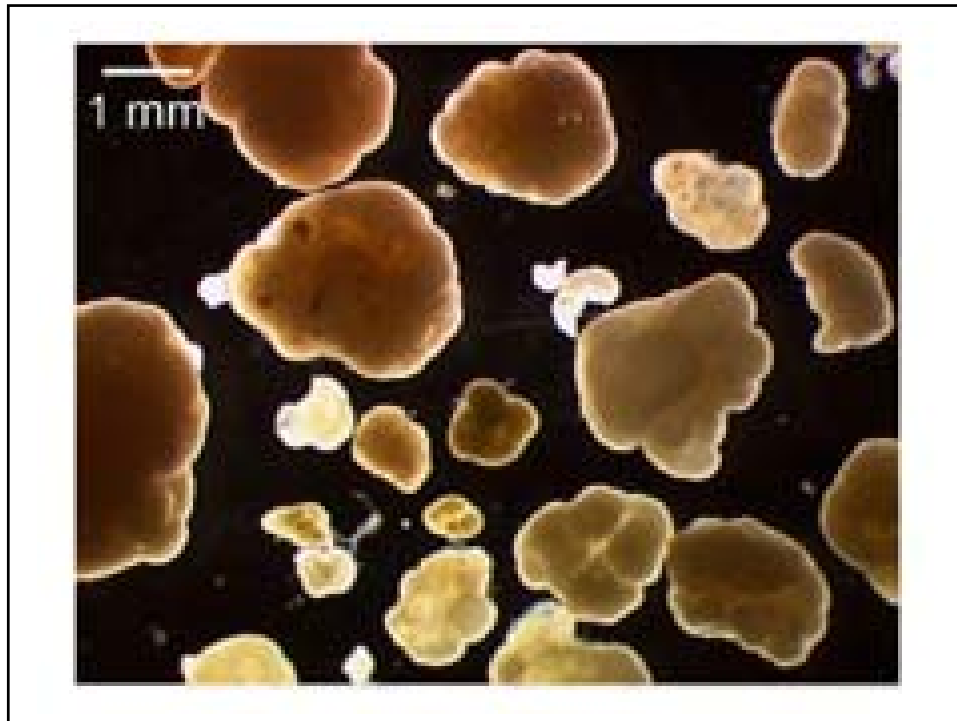
- Emerging technologies: aerobic granular sludge



Examples of settling particles

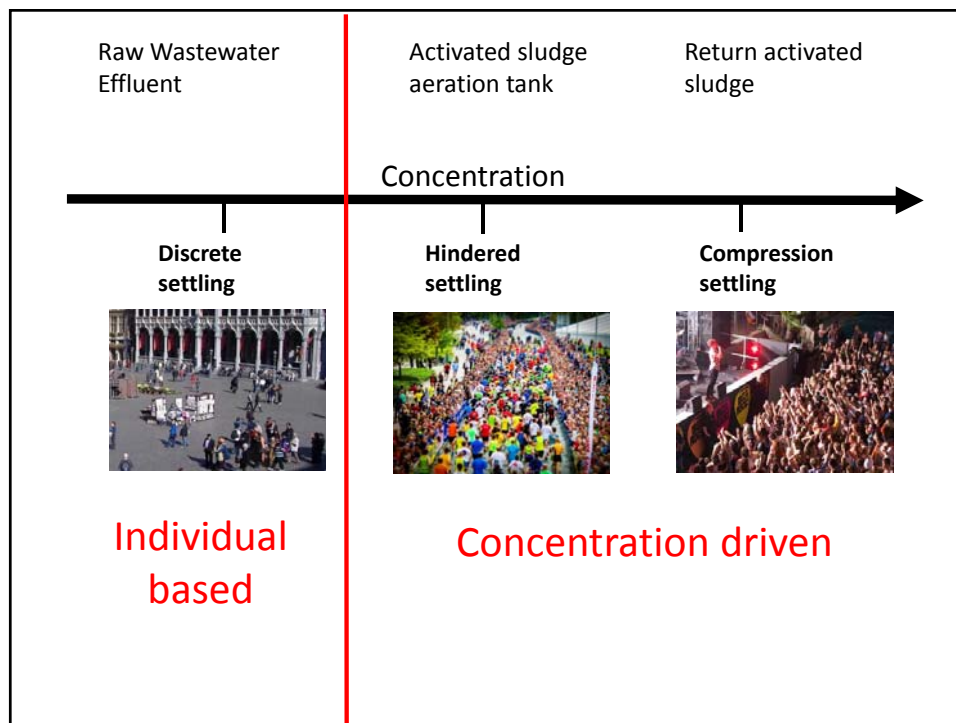
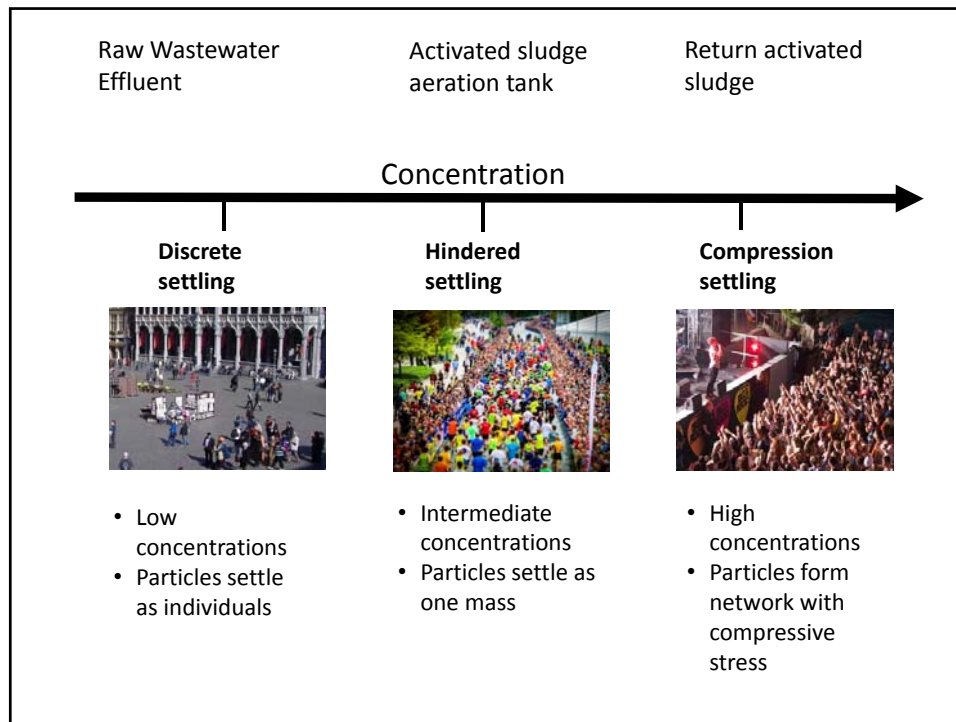






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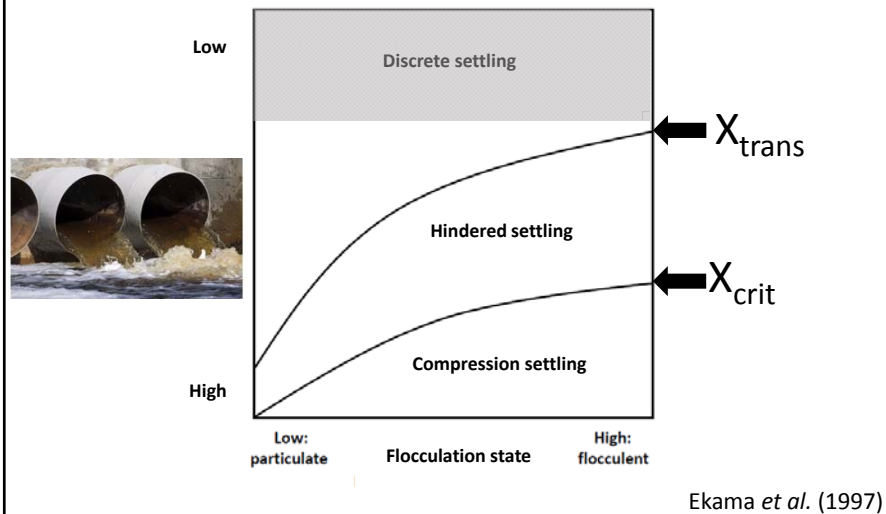
Variability in settling behaviour

Settling in different unit processes characterized by

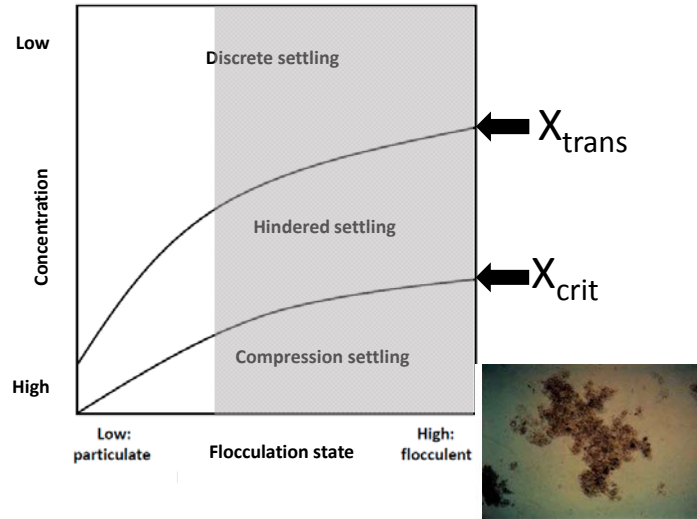
1. Differences in concentration
2. Different components with distributions in size, density, porosity,...
3. Composition changes with operating conditions (SRT, load) and environmental conditions (T)



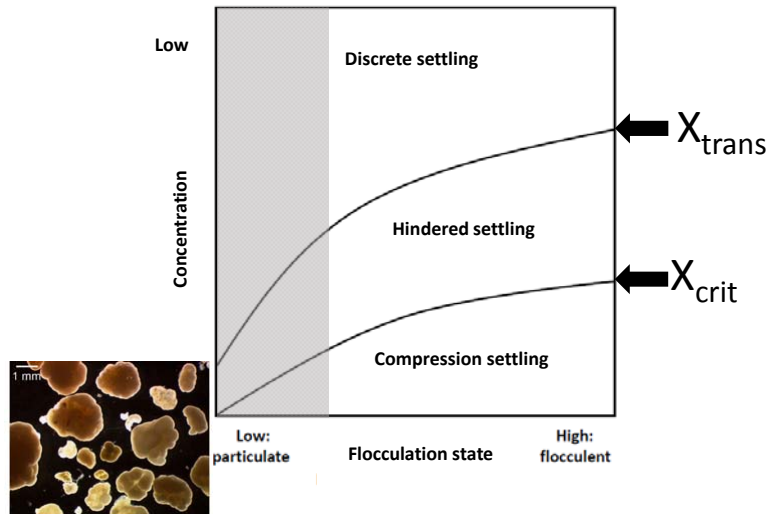
Settling behaviour



Settling behaviour



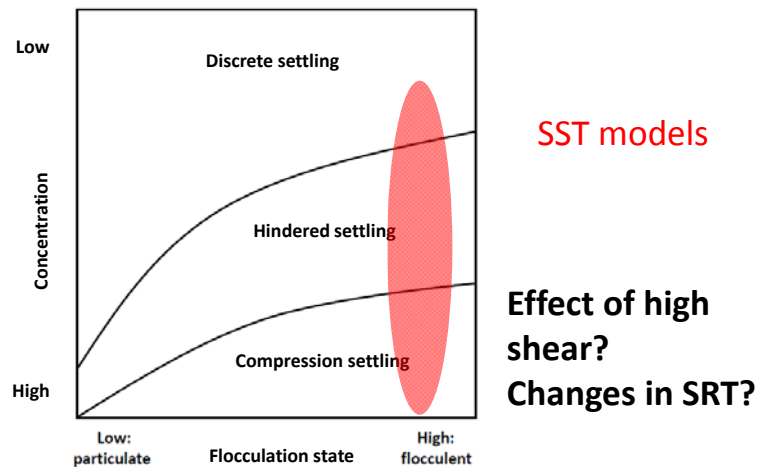
Settling behaviour



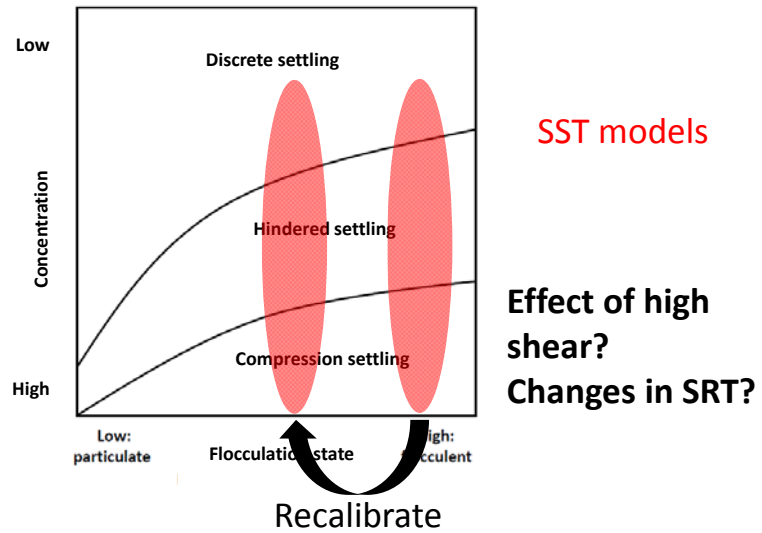
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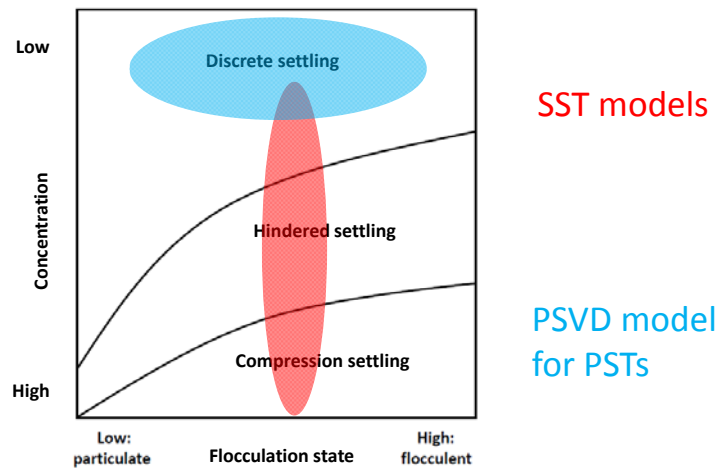
Available settling models



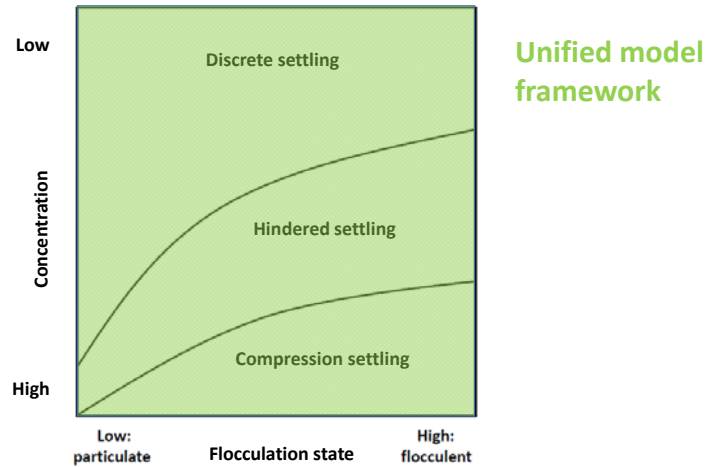
Available settling models



Available settling models



Towards a unified settling framework



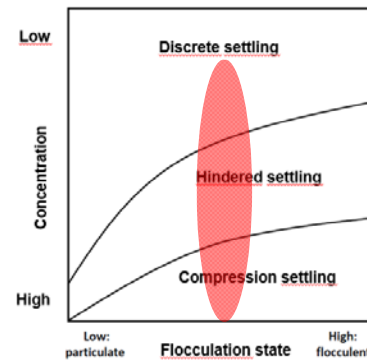
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Towards a unified settling framework

Starting point

SST model with hindered and compression settling (Bürger et al. 2013)



Extension

1. Introduce particle distribution
2. Include discrete settling behaviour
3. Couple different settling mechanisms

Step 1: Introduce distribution

- Define a number of classes i

$$X = \sum_{i=1}^n X_i$$

Class 1 Class 2 Class 3 Class 4

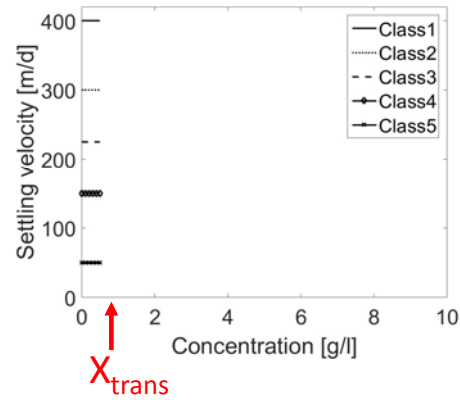
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- Classes based on size, density, settling velocity

Step 2: Discrete settling behaviour

- Discrete settling

$$V_s = V_{s,i} \text{ for each class } i$$

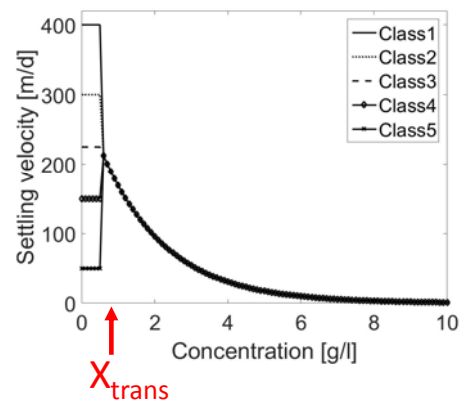


Step 3: coupling of settling behaviour

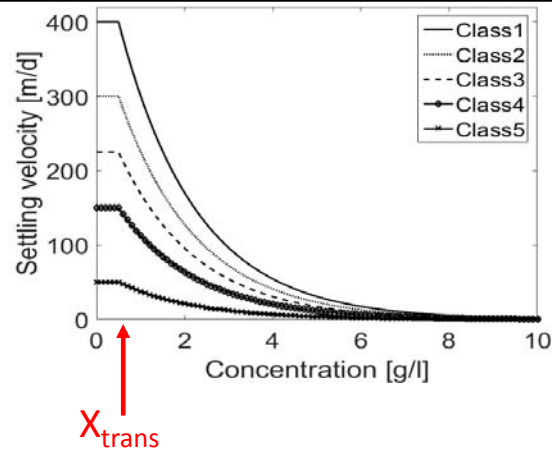
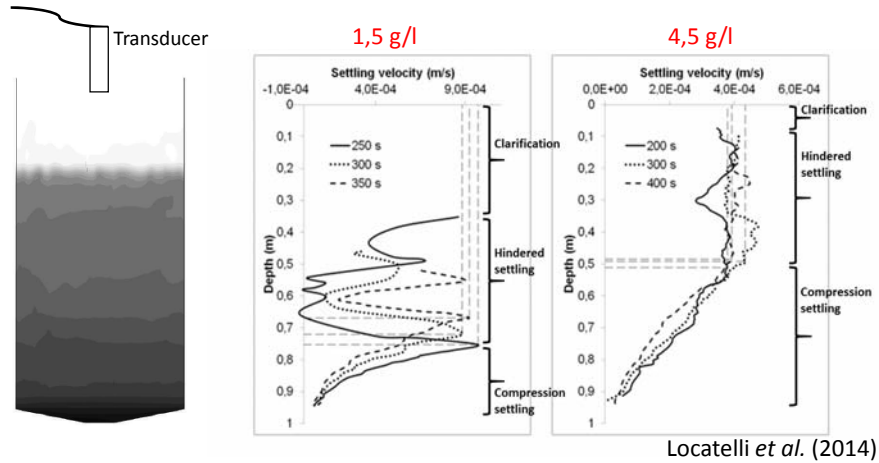
- Discrete settling

$$V_s = V_{s,i} \text{ for each class } i$$

- Coupling to hindered settling?

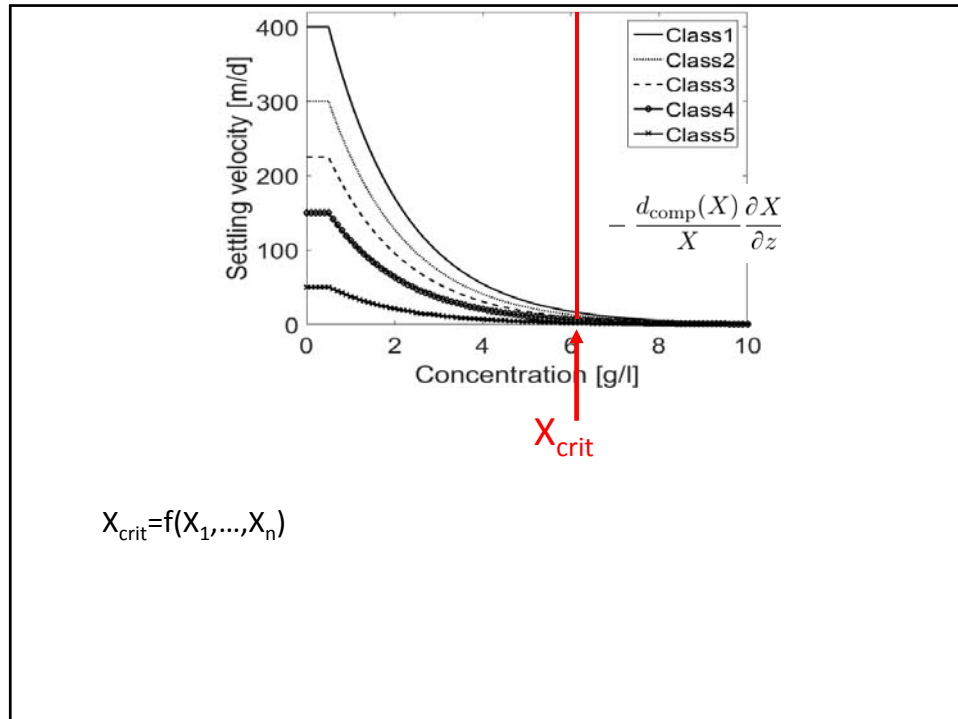


Distributed dynamics in hindered settling Velocity profile measurements



Aerobic granular sludge/Primary settling: $X_{trans} \gg \gg$

Secondary settling: $X_{trans} = f(X_1, \dots, X_n)$



Unified model equations

$$\frac{\partial X_i}{\partial t} = -\frac{\partial}{\partial z} (v_c(t) X_i) \quad \text{convective flow}$$

$$-\frac{\partial}{\partial z} (v_{ds,i}(X) X_i) \quad \text{distributed discrete and hindered settling}$$

$$+\frac{\partial}{\partial z} \left(\frac{d_{comp}(X)}{X} X_i \frac{\partial X}{\partial z} \right) \quad \text{compression settling}$$

$$+\frac{Q_f(t) X_f(t)}{A} \delta(z) \quad \text{incoming feed flow}$$

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Numerical solution

Batch settling:

- No incoming or outgoing flows

$$\frac{\partial X_i}{\partial t} = -\frac{\partial}{\partial z} (v_{as,i}(X) X_i) + \frac{\partial}{\partial z} \left(\frac{d_{\text{comp}}(X)}{X} X_i \frac{\partial X}{\partial z} \right)$$

distributed discrete
and hindered settling

compression settling

- Set of PDEs

Spatial discretisation

- Layer model
- Set of ODEs in each layer



Time discretisation

$$\frac{\partial X_i}{\partial t} + \frac{\partial}{\partial z} (v_{dhs,i}(\mathbf{X}) X_i) = \frac{\partial}{\partial z} \left(\frac{d_{comp}(X)}{X} X_i \frac{\partial X}{\partial z} \right)$$

↓

$$\partial_t \mathbf{X} + \partial_z f(\mathbf{X}) = \partial_z (B(\mathbf{X}) \partial_z \mathbf{X})$$

Numerical solution

$$\frac{\partial X_i}{\partial t} + \frac{\partial}{\partial z} (v_{dhs,i}(\mathbf{X})X_i) = \frac{\partial}{\partial z} \left(\frac{d_{comp}(X)}{X} X_i \frac{\partial X}{\partial z} \right)$$

$$\partial_t \mathbf{X} + \partial_z f(\mathbf{X}) = \partial_z (B(\mathbf{X}) \partial_z \mathbf{X})$$

Convective term:

- non-linear, gives rise to sharp discontinuities
- requires high resolution shock capturing scheme (ex. WENO)

Numerical solution

$$\frac{\partial X_i}{\partial t} + \frac{\partial}{\partial z} (v_{dhs,i}(\mathbf{X})X_i) = \frac{\partial}{\partial z} \left(\frac{d_{comp}(X)}{X} X_i \frac{\partial X}{\partial z} \right)$$

$$\partial_t \mathbf{X} + \partial_z f(\mathbf{X}) = \partial_z (B(\mathbf{X}) \partial_z \mathbf{X})$$

Diffusive term:

2nd order

Severe restrictions on max timestep: $\max \Delta t \approx \Delta z^2$

Numerical solution

- LI-IMEX scheme

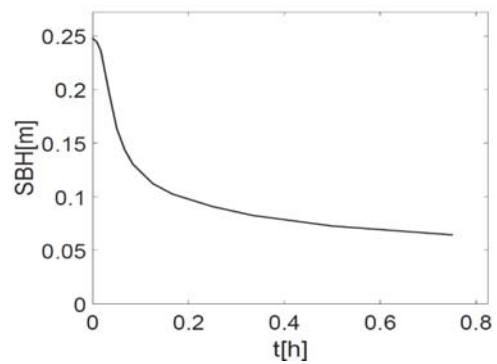
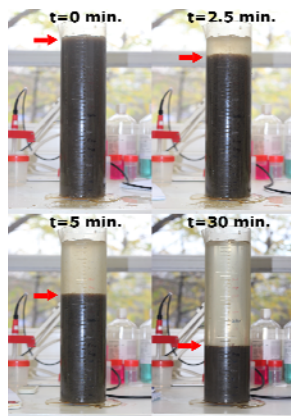
$$\mathbf{X}^{n+1} = \mathbf{X}^n - \frac{\Delta t}{\Delta z} \Delta \hat{f}(\mathbf{X}^n) + \frac{\Delta t}{(\Delta z)^2} \mathcal{B}(\mathbf{X}^n) \mathbf{X}^{n+1}$$

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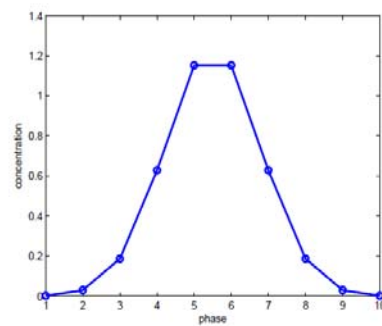
Simulation example

Batch settling experiment: measure evolution of sludge blanket height (SBH) in function of time

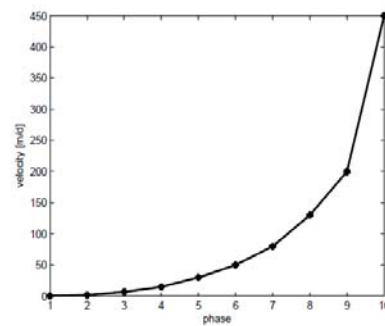


Simulation example

- Batch settling
- 3 settling regimes (discrete, hindered, compression)
- 10 classes

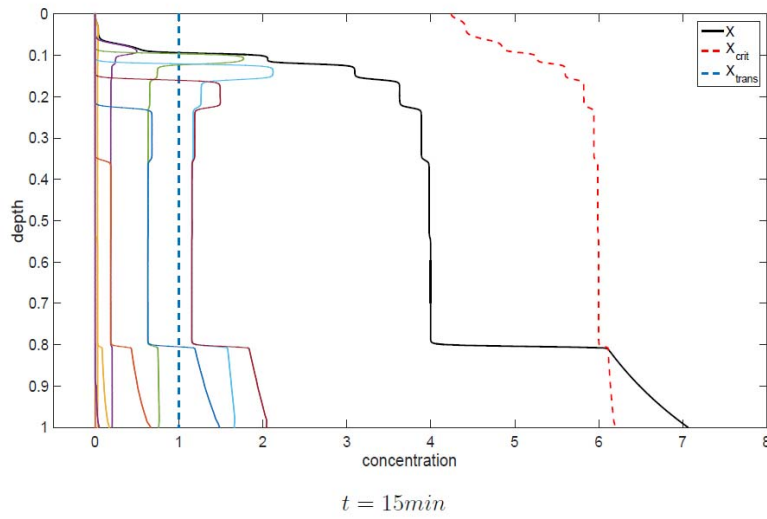


Initial concentration distribution [kg/m³]

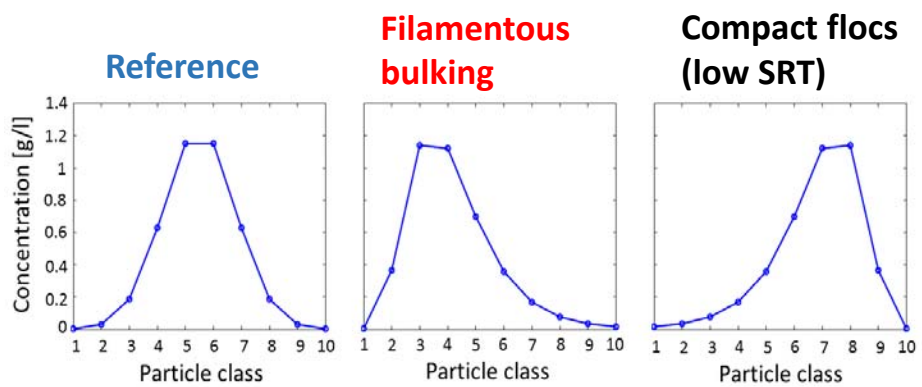


Initial velocities [m/d]

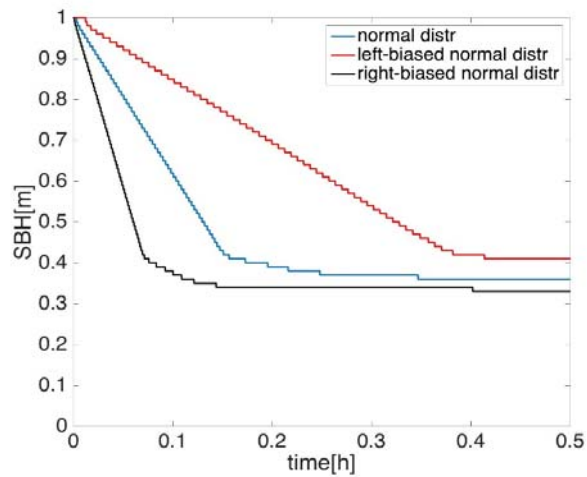
Simulation example



Simulation example



Simulation example



Conclusions

- Concentration alone is not sufficient to describe settling dynamics
- Unified framework proposed
 - Models discrete, hindered and compression settling
 - Can be applied to different settling unit processes
 - Describes effect of changes in sludge/WW composition on settling dynamics
- Numerical solution
 - Accurate and fast solutions achieved through IMEX scheme
 - High-resolution shock capturing scheme for convective term
 - Implicit scheme for the diffusive term