

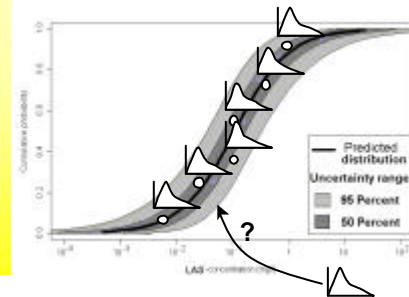
Frederik A.M. Verdonck, Olivier Thas and Peter A. Vanrolleghem

Ghent University - BIOMATH, Department of Applied Mathematics, Biometrics and Process Control, Coupure Links 653, B-9000 Ghent, BELGIUM


## Introduction (theory)

Complex data have several forms of **variations**:

- $X_1, X_2, \dots, X_n$  are i.i.d. as  $X$  ( — )
- sampling error : confidence bands around the cumulative distribution function (CDF) of  $X$  (using bootstrapping). ( — )
- $X_i$  is a summary statistic, it basically is also a random variable and  $X_2, \dots, X_n$  are also random variables.  
E.g.  $X_1$  is the mean of  $X_{11}, X_{12}, \dots, X_{1m_1}$  ( ○ )



## Introduction (case study)

In the case study,  $X_i$  is the toxicity of a chemical towards a species. 

Same forms of **variations**:

- variability between species (= inter-species sensitivity towards a chemical) ( — )
- uncertainty: sampling error ( — )
- $X_i$  is the mean of several values found in literature (from inter-laboratory variations) ( ○ )

**Goal:** How to include inter-laboratory variations?

## Proposed methodology

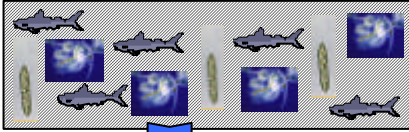
The **parametric bootstrap** method (assuming lognormal distribution) was selected as technique for characterising confidence intervals.

The answer on the question depends on the interpretation of the inter-laboratory variations: **variability** or **uncertainty**?

**Variability:** real variations, cannot be reduced through additional measurements  
=> number of samples per shot =  $m_1 + m_2 + \dots + m_n$  samples  
Two sampling strategies were investigated (same level or hierarchical level)

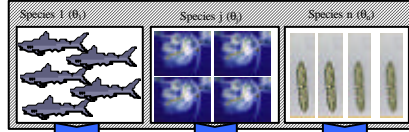
**Uncertainty:** error or ignorance, can partly be reduced through additional measurements  
=> number of samples per shot = 1

method 1: sample from entire pool



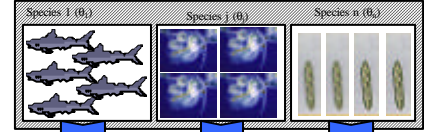
$m_1 + m_2 + \dots + m_n$  samples

method 2: sample per pool



$m_1$        $m_2$       ...       $m_n$

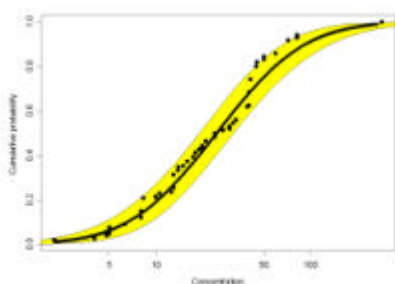
method 3



1      1      1

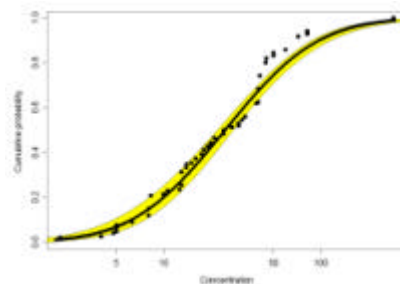
## Results + Discussion

Depending on the method used, the interpretation of the black line and its uncertainty band is different:



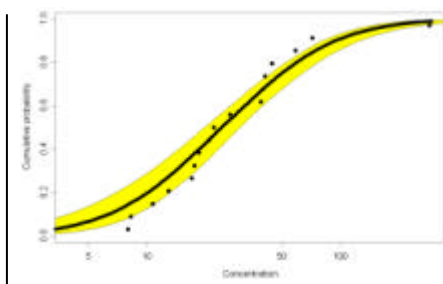
**black line** = inter-species + inter-laboratory variability  
**yellow band** = sampling uncertainty

**Interpretation:** error due to sampling between all data i.e. from the entire pool



inter-species + inter-laboratory variability  
sampling uncertainty

**Interpretation:** integrated sampling error of each species separately (i.e. between individuals per species conditioned on the species)



inter-species variability  
sampling + inter-laboratory uncertainty

Based on expert knowledge, inter-laboratory variations should be interpreted as variability because the variations are not reducible (uncertainty can always partly be reduced).

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## TAKE HOME MESSAGE

- Treating all variations on the same level (method 1) was found to be the best method for environmental standard setting because:
  - inter-laboratory variations are interpreted as variability
  - the modelled uncertainty is sampling error for all data