Modelling a nutrient deficient wastewater treatment process

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Abstract
In this paper a new model, tailor-made for nutrient deficient aerobic COD removal, is presented. The Activated Sludge Model No. 1 is chosen as a basis for model development. The original model is extended to include (1) phosphorus state variables; (2) phosphorus and nitrogen limited heterotrophic growth and (3) an additional active biomass component to model predation by higher-order organisms, e.g. protozoa. The phosphorus variables are included to model phosphorus consumption in biological growth. Nutrient limitations are modelled using Monod functions. The conventional heterotrophic biomass is assumed to be the only source of carbon and nutrients for the higher-order organisms. The model is reduced in the sense that it only regards aerobic conditions and it omits autotrophic organisms. A measurement campaign carried out at the wastewater treatment plant at Hylte mill in Sweden is used for model validation and influent fractionation. The basis for this plant design is a combined biofilm/activated sludge process. In the biofilm stage dispersed bacteria are produced, which are consumed by protozoa in the subsequent activated sludge stage. The significant presence of predating protozoa implies a negative observed yield coefficient for the activated sludge stage. An example of how the model can be used to determine operational parameters, such as sludge retention time and nutrient dosage, is shown.

Keywords
Activated sludge; dosage control; modelling; nutrient deficiency; protozoa; pulp and paper

INTRODUCTION

Being one of the major consumers of natural resources (wood, water) and energy (fossil fuels, electricity), pulp and paper production leads to significant impacts on the environment. In terms of freshwater withdrawal, the pulp and paper industry ranks third in the world, after the primary metals and the chemical industries (Thompson *et al.*, 2001). If not efficiently treated, the effluent mill water will contain large amounts of organic matter promoting microbial growth and oxygen consumption in the recipient. Wastewater from pulp and paper mills is generally treated in external biological wastewater treatment plants located at the industrial sites. The treatment typically involves aeration in ponds or tanks to remove soluble organic material and sedimentation to remove particulate matter. During the last decades, much effort has been aimed at decreasing the usage of fresh water in paper mills. This has been achieved by increased recycling of the mill process water (or whitewater) at different positions in the paper machine. There are several incentives for increasing the degree of closure of whitewater system, e.g. less dependency of access to freshwater and decreased impact on the environment. However, a high degree of closure requires internal or in-mill treatment of the whitewater. In-mill wastewater treatment (WWT) may be carried out by physical and chemical means, but biological treatment is also regarded as a possible solution. Regardless of whether the treatment is internal or external, biological treatment processes require a balanced composition of the wastewater, in particular with regard to biodegradable organic matter, nitrogen and phosphorus. Normally, the mill wastewater concentrations of nitrogen and phosphorus are low, especially in the readily available forms of ammonium and orthophosphate. To obtain efficient biological treatment, these have to be added.

There are several reasons to pursuit a robust and reliable control strategy for the nutrient dosage. Insufficient dosage will lead to decreased removal of organic components. Also, nutrient deficiency

can cause sludge bulking, which will result in operational complications and high concentrations of particulate matter in the discharged water. Overdosing, on the other hand, is undesirable for several reasons: discharge permits often specify maximum concentrations on nutrients in the effluent water, excessive dosing is costly and, if in-mill treatment is concerned, recycling nutrients to the paper making process may cause biological fouling in the paper machine. The control strategy must, consequently, handle the delicate balance between under and over dosing while ensuring high treatment efficiency. This balance is generally obtained at nutrient levels that are much lower than those encountered in, for instance, treatment of municipal wastewater. In fact, to achieve the goal of only small fractions of nutrients in the effluent, the processes must be operated under what is considered as nutrient deficient conditions. Since the organic load on the treatment process is varying, robust control of nutrient dosage is far from trivial and information on the biological process under the present conditions is imperative.

When determining the nutrient requirements of a nitrogen and phosphorus deficient biological treatment process, three operational parameters are of great interest: 1) the organic load; 2) the observed yield coefficient, i.e. the amount of produced sludge and 3) the nutrient content of the sludge. To assess how these parameters influence the process and its control, a model of the process is valuable. Modelling of WWT processes has been an area for research during the last decades. Today, a number of models for different biological treatment processes are available, of which the models prepared by Task Groups under the International Water Association (IWA) umbrella are the most widespread (see Henze *et al.*, 2000). They have, however, primarily been adapted to municipal wastewater treatment characterised by relatively high concentrations of nitrogen and phosphorus. Hence, the most commonly used of these models, the Activated Sludge Model No. 1 (ASM1) does not describe nutrient limitations or phosphorus. The successor of ASM1, i.e. ASM2d, does model nutrient limitations and particularly phosphorus behaviour, but is considered to be overly complicated for the application under study since it is designed to model the specific metabolism of enhanced biological phosphorus removal. ASM3 describes nitrogen limitation but it does not model phosphorus. Consequently, for the purpose of modelling treatment of pulp and paper mill wastewater, a new model must be developed.

In this work, we describe the development of a model that is tailor-made for aerobic treatment of pulp and paper wastewater. We have chosen ASM1 as a basis for the model development. There are some important requirements on the new model: (i) the model should be able to describe nutrient limited growth of heterotrophic biomass; (ii) the model only needs to model aerobic processes; (iii) the model should describe the growth of higher-order organisms and their interaction with the heterotrophic biomass (note, in this work, higher-order organisms are not included in the heterotrophic biomass). The reasons for (i) and (ii) has been discussed above. The reason for (iii) is that treatment technologies have appeared that utilise the fact that the observed yield can be significantly lowered by exploiting the higher-order organisms, e.g. protozoa, within the process (Ratsak *et al.*, 1996; Welander *et al*., 2002).

MODEL DEVELOPMENT

To accurately model nutrient deficient conditions using ASM1, states and processes describing phosphorus in the system must be added. Moreover, a significant appearance of protozoa in activated sludge plants reduces sludge production, an important phenomenon associated with nutrient regeneration. An extra state variable is therefore included to model the biomass of higher-order organisms. At present, the new model describes only aerobic environments and anoxic growth is not considered. An assumed absence of nitrate/nitrite justifies neglecting the autotrophic biomass and the associated autotrophic growth and decay processes. Consequently, the model proposed in this paper is on the one hand an extended, and on the other hand, a reduced version of ASM1. Below, only the major extensions that have been made to ASM1 are discussed and it is

presupposed that the reader is familiar with the original model. In the appendix, the modified ASM1 is presented using the traditional matrix representation. For simplicity, inert components originating from influents are not shown.

State variables

Phosphorus (P) occurs in wastewater partly as organically bound (particulate and soluble) P, and partly as inorganic (soluble) P. In biological WWT processes, a majority of the various P compounds is hydrolysed to form orthophosphate, which is the fraction available for heterotrophic growth (Henze *et al.,* 1995). In model terms, the total P is divided into soluble P, particulate organic P and active biomass P. The soluble non-inert form consists of inorganic P and soluble organically bound P. It is assumed that all (non-inert) soluble P is hydrolysed into orthophosphate rapidly, in other words, that it becomes available for growth in a short time span. With this assumption, all non-inert soluble P is readily available for assimilation and it is lumped into one state variable (S_P). Particulate organic P is divided into degradable and non-biodegradable particulate P. The degradable fraction is made up of particulate organically bound P (X_{PD}) while the nondegradable fraction consists of particulate bound P arising from biomass decay (X_{PP}). Active biomass P, incorporated into both floc-formers ($X_{B,H}$) and protozoa ($X_{B,M}$), is denoted X_{PB} (see Figure 1a). The carbonaceous and nitrogenous materials are divided similarly to the fractionations presented in ASM1 (see Figures 1b and 1c). However, an additional COD state variable, to model the active protozoan biomass ($X_{B,M}$), has been added.

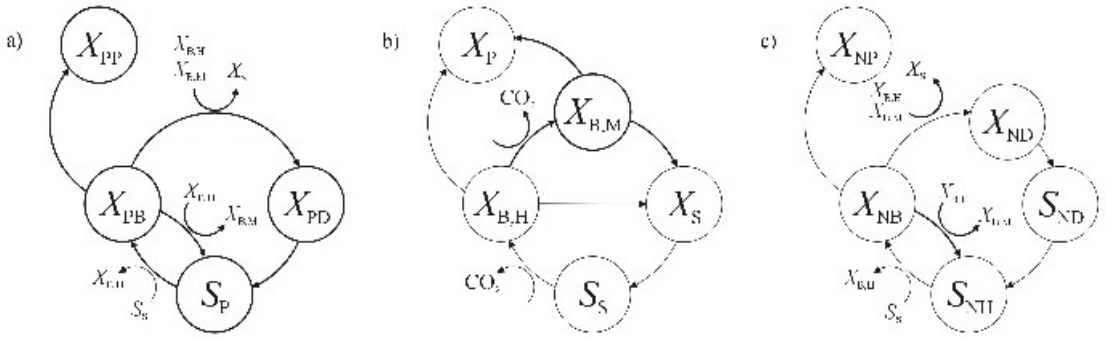

Figure 1. The fractionations and transformations of phosphorus (a), carbonaceous (b) and nitrogenous (c) state variables in the extended model. Notations are according to Henze *et al.* (2000) where not else notified. State variables and transformation arrows given in bold denote extensions that have been made to ASM1. Inert components originating from influents are not shown.

Process kinetics

Some kinetic expressions (denoted ρ_j in Appendix 1) in the modified model equal the ones presented in ASM1. These include decay of heterotrophs (ρ_2), ammonification of soluble organic nitrogen (ρ_3), hydrolysis of entrapped organics (ρ_4) and the corresponding hydrolysis of entrapped organic nitrogen (ρ_5). The inclusion of phosphorus state variables allows for the addition of both nitrogen and phosphorus Monod limiting functions for the aerobic growth of heterotrophs (ρ_1), an approach inspired by ASM2d (Henze *et al.*, 2000). The growth rate of protozoa (ρ_7) is modelled similar to the growth rate of heterotrophs. It is assumed that the nutrient contents of the protozoa equal the contents of the heterotrophs; the growth of protozoa is therefore not limited by exogenous soluble nitrogen and phosphorus. Instead, the process is limited by the concentration of heterotrophs (the substrate for the protozoa) and oxygen, since protozoa are inactive under anoxic/anaerobic conditions (Van Loosdrecht and Henze, 1999). The limitations of protozoan growth are described with Monod functions. It is an open question whether this approach is correct or not. Ratsak *et al.* (1996) suggests that a certain amount of bacteria is required to initialise protozoan growth and the proposed process rate is one way to describe this. The decay of

protozoa (ρ_8) is modelled similar to the decay of heterotrophs (ρ_2) in ASM1. In the model, it is assumed that particulate organic phosphorus is hydrolysed (ρ_6) in the same way as entrapped organics and nitrogen (ρ_4, ρ_5). As discussed above, the immediate product of the hydrolysis is soluble P available for assimilation.

Model formulation

The dynamic behaviour of the protozoan biomass is affected by growth and decay (see Figure 1b). The growth of higher-order organisms reduces the heterotrophic biomass. During energy transfer from bacteria to protozoa, energy is lost (from the system) due to inefficient biomass conversion. This is balanced with an associated oxygen demand and CO_2 production. Like for S_{NH} in ASM1, S_P is consumed and converted into active biomass P (X_{PB}) when new heterotrophic biomass is synthesised, see Figure 1a. During heterotrophic and protozoan decay, P in products arising from biomass decay (X_{PP}) and particulate biodegradable P (X_{PD}) are formed. The reduced sludge production caused by the protozoan predation leads to an interesting phenomenon with regard to the nutrients. Consider for example the conversions of soluble phosphorus available for assimilation (S_P) in the 12th column of the model matrix. It is produced by the hydrolysis process and consumed through incorporation into heterotrophic biomass. The third process affecting the component is the predation of protozoa on the heterotrophs. The process converts only part of the heterotrophic biomass into new biomass. The phosphorus part of the heterotrophic biomass not used for synthesis of protozoan biomass will be regenerated to the solution and eventually be available for growth of bacteria.

CASE STUDY

In the fall of 2002, a new wastewater treatment plant (WWTP) was taken into operation at Hylte mill, a large producer of standard newsprint (850 000 tons/year) located in southern Sweden. The plant configuration involves a biofilm stage followed by an activated sludge (AS) stage, of which both are aerated. As for most pulp and paper WWT systems, phosphorus and nitrogen (N) supplements are required, since the influent wastewater does not contain sufficient amounts for efficient biological COD removal. Readily available P and N are added to the biofilm stage. Here, dispersed bacteria are produced, which are consumed by protozoa in the subsequent AS stage. The process principle aims at robust operation with low sludge production and good sludge settleability. A result of the plant design is that the consumption of dispersed bacteria in the AS stage implies a release of soluble nutrients. This phenomenon complicates nutrient dosage analysis and control. To determine the performance of the Hylte mill WWTP and to achieve necessary information for model input fractionation and validation, a measurement campaign were carried out at the plant.

Measurement campaign

Although the overall plant was investigated, the focus of this paper lies on the AS stage, to which the proposed model is applied. In particular, the transformations of COD and P components are considered. During three days, grab samples were taken once a day from the influent, mixed liquor and effluent of the AS stage. The samples were analysed as total and filtered with respect to COD, orthophosphate, total P, ammonia and total N using standard methods. Series of daily data received from the WWTP water laboratory confirmed that the campaign results were representative of steady-state operation. No nitrate/nitrite was detected. Some main results of the campaign are shown in Table 1.

Table 1. Mean values from the measurement campaign at the Hylte mill WWTP AS stage.

Parameter	Unit	Influent AS	Mixed liquor	Effluent	Underflow	Parameter	Unit	AS stage
Soluble COD	mg COD/l	1233	330	310	-	Sludge production	kg X_{COD}/d	-2000
Particulate COD	mg COD/l	656	6300	20	11560	Obs.yield. coeff.	kg S_{COD}/kg X_{COD}	-0.12
Soluble phosphorus	mg P/l	0.40	0.23	0.57	-	Sludge retention time	d	9.4
Orthophosphate	mg P/l	0.15	0.15	0.27	-	Influent flow rate	m^3/d	18 200
Particulate phosphorus	mg P/l	2.35	24.2	<0.1	46	Excess sludge flow rate	m^3/d	860

Model parameter identification

The results from the measurement campaign, together with experiences from earlier studies on pulp and paper mill wastewater (Alexandersson *et al.*, 2003), are used to adapt the model parameter set and the influent fractionation to the conditions of the Hylte mill WWTP. However, when no experimental information is available, parameter values for municipal wastewater (compensated for a temperature of 30 °C using an Arrhenius equation) from the ASM1 and ASM2d model formulations are used throughout the model validation and simulations. Besides many of the parameters included in ASM1, a number have been added to the extended model. These are, together with parameter values that differ from Henze *et al.* (2000), listed in Table 2. Note that parameters not listed in the table are according to ASM1.

Table 2. A summary of new (bold) and modified parameters. *Alexandersson *et al.* (2003) suggest that K_P is not higher than 0.05 and that K_{NH} is not higher than 0.1. Note, parameters not listed in the table are according to ASM1 (30°C).

Extended model parameters	Symbol	Unit	Used value	ASM1/ASM2d (30°C)
Stoichiometric coefficients				
Protozoan yield	$\boldsymbol{Y_M}$	**g cell COD formed (g cell COD oxidised)$^{-1}$**	**0.335**	-/-
Fraction of protozoan biomass yielding particulate products	$\boldsymbol{f_{PM}}$	**dimensionless**	**0.08**	-/-
Mass N/mass COD in active biomass	i_{XB}	g N (g COD)$^{-1}$	0.05	0.086/0.07
Mass N/mass COD in products from biomass decay	i_{XP}	g N (g COD)$^{-1}$	0.035	0.06/0.02
Mass N/mass COD in inert particulate matter	i_{XI}	g N (g COD)$^{-1}$	0.04	-/0.02
Mass P/mass COD in active biomass	i_{XBP}	g P (g COD)$^{-1}$	0.005	-/0.02
Mass P/mass COD in products from biomass decay	i_{XPP}	g P (g COD)$^{-1}$	0.004	-/0.01
Mass P/mass COD in inert particulate matter	i_{XIP}	g P (g COD)$^{-1}$	0.004	-/0.01
Kinetic parameters				
Protozoan max. specific growth rate	$\boldsymbol{\mu_{max,M}}$	**day^{-1}**	**1.2**	-/-
Decay rate coefficient for protozoan organisms based on the death-regeneration hypothesis	$\boldsymbol{b_M}$	**day^{-1}**	**0.1488**	-/-
Half-saturation constant for heterotrophs	$\boldsymbol{K_{XBH}}$	**mg COD/l**	**7000**	-/-
Half-saturation constant for oxygen	$\boldsymbol{K_{O,M}}$	**mg O_2/l**	**0.2**	-/-
Decay rate coefficient for heterotrophic organisms based on the death-regeneration hypothesis	b_H	day^{-1}	0.744	1.86/0.8
Half-saturation constant for soluble phosphorus	K_P	mg P/l	0.05*	-/0.01
Half-saturation constant for ammonia nitrogen	K_{NH}	mg N/l	0.1*	-/0.05

Despite presence of protozoa, the mixed liquor suspended solids (MLSS) concentration is surprisingly high, averaging at 6300 mg COD/l. According to microscopy studies of the actual sludge composition, a majority of this is made up of biomass; there does not seem to be an abundance of inert particulate matter. To maintain such a high MLSS concentration, the decay rate of the model must be lowered. This is explained by the following reasoning. Disappearance of suspended organic matter can be a result of numerous mechanisms: maintenance energy requirements, lysis of cells, endogenous respiration and grazing by higher-order organisms. In ASM1 these internal and external processes are, according to the death-regeneration hypothesis, lumped together and described by the temperature-dependent decay process (Van Loosdrecht and Henze, 1999). Here the protozoan activity, which contributes significantly to the external decay rate, is separated from the above decay process and modelled explicitly as growth of higher-order organisms . As a consequence, the default decay rate for heterotrophs given in ASM1 is lowered.

Although important, the role of protozoa in AS processes seems to be an orphan in the academic research (Van Loosdrecht and Henze, 1999) and no kinetic parameters for the protozoa have been found in the literature. Instead, these are chosen so that the protozoan population represents what the microscopy studies suggest, that is, about 10-20% of the MLSS concentration. This is achieved by setting the growth and decay rates to about 10% of those of the heterotrophs and by choosing a high half-saturation constant. The soluble organic load into the AS stage is 22 400 kg S_{COD}/d of which, 76% is removed. The flux of sludge from the secondary settler underflow is approximately 9900 kg X_{COD}/d. The flux of particulate material into the AS stage (to a significant extent made up of dispersed bacteria, produced in the foregoing biofilm stage) is 11 900 kg X_{COD}/d. If this influent flux is deducted from the traditional sludge production (SP) calculation, we conclude that

SP = −2000 kg X_{COD}/d, i.e. the predation of protozoa on the influent dispersed bacteria implies a negative observed yield coefficient for the AS process averaging −0.12 kg X_{COD}/S_{COD}. By adopting a protozoan yield of 0.335 kg $X_{B,M}/X_{B,H}$ this net sludge degradation is achieved by the model.

The mean nutrient content of the particulate matter averages 4% N and 0.4% P (on a COD basis), a relatively poor nutrient content typical for pulp and paper wastewater. Despite the transformation of dispersed bacteria into floc-forming bacteria and protozoa throughout the AS stage, the nutrient to COD ratios do not vary. As seen in Table 2, the mass of nutrients per mass of COD in the various particulate components (denoted *i*), are therefore set relatively similar.

The model is calibrated for steady-state conditions. Still, it should be mentioned that several different parameter sets, giving rise to similar model behaviour, exist. Future dynamic validation of the model will help us to reduce the number of feasible parameter sets.

Influence of operating conditions

From practical experience, it is well known that the sludge production of a WWTP decreases with increasing sludge retention time (SRT). With a large fraction of protozoa (which is the case at the Hylte mill WWTP), this becomes more evident. The required addition of nutrients is intimately coupled to the sludge production and thus, it is indirectly a function of the SRT. The new model can be used as guidance in choosing an appropriate SRT as well as an adequate nutrient dosage.

Steady-state solutions of numerous simulations are used to investigate the influence of various SRT and nutrient addition values on the process. Since effluent phosphorus seems to be more difficult to control than effluent nitrogen at the Hylte mill WWTP, only P-limitation is considered. The measurement results are used to determine the characteristics of the influent wastewater to the AS stage in the simulations. The influent soluble P is controlled so that all soluble biodegradable COD is reduced. Two cases are studied: 1) all influent particulate P is biodegradable and 2) only 80 % of the influent particulate P is biodegradable. The reason for looking at these two cases is that the availability of P incorporated in the influent dispersed bacteria has a great impact on the choice of SRT.

The results of the simulations are shown in Figure 2. It can be seen from the figure (left) that an increasing SRT favours protozoa at the expense of heterotrophs. At higher SRTs, the operating conditions allow the protozoan organisms to grow and compete with the heterotrophs, while at lower SRTs, they tend to be washed out. From the right plot, the difference between the two cases is clear. Thus, the biodegradability of particulate P in the influent of the activated sludge stage must be investigated to develop a robust nutrient dosage control strategy. This is an evident example on how influent fractionation and parameter estimation determine the model outputs.

From Figure 2 (right) it can also be seen that a minimum in required P-addition and effluent soluble P coincide when the observed yield of the AS stage is zero, i.e. at SRT=5.4 and SRT=7.9 days for the two cases, respectively. This does not mean that a SRT that results in a minimum of dosage and effluent P is the most desirable choice. In fact, several other factors have to be considered. A low SRT results in a minimum of effluent P, low costs for aeration, but high sludge production and costs for nutrient addition. On the other hand, a high SRT will give lower sludge production, lower (or no) nutrient addition costs, but higher concentrations of effluent P, increased aeration costs and potential degeneration of sludge quality. Thus, finding a suitable SRT and nutrient dosage is a matter of weighing a number of operational criteria so that economical and environmental requirements are fulfilled.

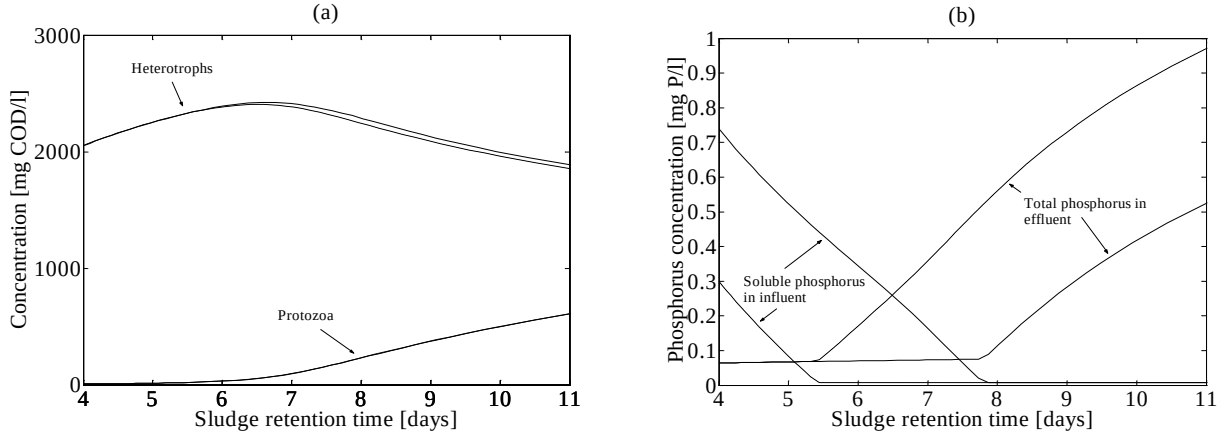

Figure 2. Steady-state solutions for simulations with different SRTs. Continuous lines: All phosphorus biodegradable. Dotted lines: 80% of particulate phosphorus biodegradable.

CONCLUSIONS

A new model, tailor-made for nutrient deficient aerobic COD removal is presented. It is based on ASM1 extended with additional states to model the role of phosphorus in biological growth and higher-order organisms. Moreover, nutrient limitations are included in the model by adding Monod functions to the heterotrophic growth expression. The model has been successfully calibrated for steady-state conditions using the results from a measurement campaign carried out at the Hylte mill WWTP. The identified model parameter values do not deviate significantly from those found in the literature, except for the decay coefficient for heterotrophic biomass. The deviation is explained by interactions between the conventional heterotrophic biomass and the new state describing higher-order organisms. Simulations of the Hylte mill WWTP using the new model show that the biodegradability of particulate phosphorus in the influent to the activated sludge stage is an important piece of information that must be further investigated. The model is used to find appropriate operating conditions in terms of SRT and nutrient dosage. The result indicates that an optimal SRT can be found if only the phosphorus addition and the effluent phosphorus concentration are concerned. However, in the determination of an appropriate SRT, other factors must be considered. Finding a suitable SRT and nutrient dosage is a matter of weighing a number of operational criteria so that economical and environmental requirements are fulfilled.

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APPENDIX. The extended model matrix.

Table A.1. The extended model matrix. Notations in bold refer to coefficients, state variables and processes that have been added to ASM1.

i	Component →	1.	2.	3.	**4.**	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	Process Rate ϱ_j
j	Process ↓	S_S	X_S	X_{BH}	$\boldsymbol{X_{BM}}$	X_P	S_O	S_{NH}	S_{ND}	X_{ND}	X_{NB}	X_{NP}	$\boldsymbol{S_P}$	$\boldsymbol{X_{PD}}$	$\boldsymbol{X_{PP}}$	$\boldsymbol{X_{PB}}$	↓
1.	Aerobic growth of heterotrophs	$-\frac{1}{Y_H}$		1			$-\frac{1-Y_H}{Y_H}$	$-i_{XB}$			i_{XB}		$\mathbf{-i_{XBP}}$			$\mathbf{i_{XBP}}$	$\hat{\mu}_H\left(\frac{\mathbf{S_{NH}}}{\mathbf{K_{NH}+S_{NH}}}\right)\left(\frac{\mathbf{S_P}}{\mathbf{K_P+S_P}}\right)\left(\frac{S_S}{K_S+S_S}\right)\cdot\left(\frac{S_O}{K_{O,H}+S_O}\right)X_{B,H}$
2.	Decay of heterotrophs		$1-f_P$	-1		f_P				$i_{XB}-f_P\cdot i_{XP}$	$-i_{XB}$	$f_P\cdot i_{XP}$		$\mathbf{i_{XBP}-f_P\cdot i_{XPP}}$	$\mathbf{f_P\cdot i_{XPP}}$	$\mathbf{-i_{XBP}}$	$b_H X_{B,H}$
3.	Ammonification of soluble organic nitrogen							1	-1								$k_a S_{ND} X_{B,H}$
4.	Hydrolysis of entrapped organics	1	-1														$k_h\frac{X_S/X_{B,H}}{K_X+(X_S/X_{B,H})}\left(\frac{S_O}{K_{O,H}+S_O}\right)X_{B,H}$
5.	Hydrolysis of entrapped organic nitrogen								1	-1							$k_h\frac{X_S/X_{B,H}}{K_X+(X_S/X_{B,H})}\left(\frac{S_O}{K_{O,H}+S_O}\right)\frac{X_{ND}}{X_S}X_{B,H}$
6.	**Hydrolysis of entrapped organic phosphorus**												**1**	-1			$\mathbf{k_h}\frac{\mathbf{X_S/X_{B,H}}}{\mathbf{K_X+(X_S/X_{B,H})}}\left(\frac{\mathbf{S_O}}{\mathbf{K_{O,H}+S_O}}\right)\frac{\mathbf{X_{PD}}}{\mathbf{X_S}}\mathbf{X_{B,H}}$
7.	**Aerobic growth of higher-order organisms**			$-\frac{\mathbf{1-Y_M}}{\mathbf{Y_M}}$	**1**		$-\frac{\mathbf{1-Y_M}}{\mathbf{Y_M}}$	$\mathbf{i_{XB}}\frac{\mathbf{1-Y_M}}{\mathbf{Y_M}}$			$\mathbf{-i_{XB}}\frac{\mathbf{1-Y_M}}{\mathbf{Y_M}}$		$\mathbf{i_{XBP}}\frac{\mathbf{1-Y_M}}{\mathbf{Y_M}}$			$\mathbf{-i_{XBP}}\frac{\mathbf{1-Y_M}}{\mathbf{Y_M}}$	$\hat{\mu}_M\frac{X_{B,H}}{K_{XB,H}+X_{B,H}}\left(\frac{S_O}{K_{O,M}+S_O}\right)X_{B,M}$
8.	**Decay of higher order organisms**		$\mathbf{1-f_{PM}}$		**-1**	$\mathbf{f_{PM}}$				$\mathbf{i_{XB}-f_{PM}\cdot i_{XP}}$	$\mathbf{-i_{XB}}$	$\mathbf{f_{PM}\cdot i_{XP}}$		$\mathbf{i_{XBP}-f_{PM}\cdot i_{XPP}}$	$\mathbf{f_{PM}\cdot i_{XPP}}$	$\mathbf{-i_{XBP}}$	$\mathbf{b_M X_{BM}}$
		Readily biodegradable substrate [M(COD)L^{-3}]	Slowly biodegradable substrate [M(COD)L^{-3}]	Active heterotrophic biomass [M(COD)L^{-3}]	**Active protozoan biomass [M(COD)L^{-3}]**	Particulate products arising from biomass decay [M(COD)L^{-3}]	Oxygen (negative COD) [M(-COD)L^{-3}]	$NH_4^+ + NH_3$ nitrogen [M(N)L^{-3}]	Soluble biodegradable nitrogen [M(N)L^{-3}]	Particulate biodegradable nitrogen [M(N)L^{-3}]	Active mass nitrogen [M(N)L^{-3}]	Nitrogen in products arising from biomass decay [M(N)L^{-3}]	**Soluble phosphorus available for assimilation [M(P)L^{-3}]**	**Particulate biodegradable phosphorus [M(P)L^{-3}]**	**Phosphorus in products arising from biomass decay [M(P)L^{-3}]**	**Phosphorus in active biomass [M(P)L^{-3}]**	