Use of Variance decomposition in the early stages of WWTP design

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Abstract
Variance decomposition is performed to assess how ranges of inputs contribute to the variance of the design volume of a nitrifying WWTP. Exploring both first order – or main effects and total order effects allows discriminating between influential and non-influential decisions of various stakeholders in the early phases of a design project.

Keywords
WWTP design, variance decomposition, global sensitivity analysis, interactions, Extended-FAST

INTRODUCTION

Design of a WWTP is a function of different input factors. In the initial stages of WWTP planning and design assumptions need to be made on their respective values. MC simulation together with global sensitivity analysis (GSA) methods can assist in this. Previously the Metcalf and Eddy design guidelines have been assessed using the Standardised Regression Coefficients (SRC) as the method for GSA (Flores-Alsina *et al.*, 2010). However, as this method is only valid for systems that fulfil certain assumptions (quasi-linearity) we propose the use of Extended-FAST, a more sophisticated method, to decompose the variance of the design variable into so called main- or 1st order effects S and total effects S_T (Saltelli *et al.*, 1999). The advantage of the method with respect to other GSA methods is that no assumptions on linearity, additivity or monotonicity need to be made. The drawback of the method, i.e. its high computational cost, is not limiting as the equations of design guidelines can be computed very fast. In this paper we propose to apply this method in the early stages of a WWTP design. For illustrative purposes we use a simplified version of the ATV A-131 design guideline (ATV-DVWK, 2000).

The value ranges of the different factors to which a design is sensitive, can be due to forecasting uncertainties, process engineering uncertainty, process or input variability, choices on operational settings and choices on safety factors. They can be attributed to different stakeholders in the design phase: city planner, process design engineer, operator and owner.

MATERIALS AND METHODS

Simplified Design

A WWTP is to be designed for complete nitrification throughout the year (Phosphorous removal is not required). We use a simplified version of the ATV A-131 guideline as described in Gujer

(1999). Hereby the Sludge Retention Time (SRT) is determined by a safety factor to protect for short term loading of ammonium (fNH_4) and by the critical growth rate of nitrifiers, which itself is a function of minimal temperature (T_{min}) (eq. 1). The required tank volume then becomes a function of the SRT, the sludge production $\left(USB \times Q \times BOD_{5_{in}}\right)$ and the reactor TSS concentration (eq. 2). Hereby USB is the specific sludge production (kg TSS / kg BOD_5), Q is the 85% percentile of daily flows and BOD_{5in} is the average influent BOD_5 concentration after primary settling.

$$SRT = fNH_4 / \left(0.29 \times e^{\left(0.11 \times (T_{min} - 10°C)\right)}\right) \qquad \text{eq. 1}$$

$$Volume = SRT \times \left(USB \times Q \times BOD_{5_{in}}\right) / TSS \qquad \text{eq. 2}$$

During the design stage assumptions need to be made about the values of the factors in eq. 1 and 2 which are qualitatively very different. Q and BOD_{5in} are uncertain due to the unknown future development of the load characteristics across the design horizon in combination with unknown developments of weather and climate patterns (T_{min}). Other factors exhibit variation and uncertainty due to changes in process and wastewater characteristics (USB). The safety factor fNH_4 is typically the estimated ratio between the 80% percentile of the daily 2 hour maximum NH_4 load to the average daily NH_4 load (both expressed in kg N d^{-1}). Due to uncertainty of load (dynamics) development this safety factor will be chosen with more or less conservatism. Finally, TSS is an operational parameter which will usually be determined within a certain range in the design stage depending on other considerations. The ranges are described by uniform probability distributions (Table 1).

Table 1. *Factors of simplified guideline characterised by uniform probability distributions.*

Factor	min	max	Units
Q	32000	48000	m^3d^{-1}
BOD_{5in}	0.09	0.15	g m^{-3}
TSS	2	4	kg m^{-3}
USB	0.8	1	kg TSS / kg BOD_5
fNH_4	2	3	-
T_{min}	9	12	°C

Monte Carlo Simulation

In a first step a Monte Carlo (MC) simulation is performed by random sampling from the distributions of the factors and propagation to the model output (Volume_MC). The resulting distribution is visualised by a histogram and quantitative characteristics such as mean and standard deviation (sd) are computed.

Variance Decomposition

Methods based on Variance Decomposition have several advantages over other Sensitivity Analysis methods: they can cope with the location, scale and shape factor ranges. They allow for multidimensional averaging and they are model-independent, i.e. they do not require the relationships between model outputs and input factors to be linear, monotonous, additive or continuous (Saltelli *et al.*, 2004).

The first order- or main effect sensitivity index, S, indicates which factors, if assumed known, would be expected to reduce the variance in the design variable the most ("factors prioritisation"). On the other hand the total order sensitivity index, S_T, will indicate which factors can be fixed anywhere within their range without significantly changing variance of the design variable (factors fixing). Due to interactions between the factors these two indices may differ considerably (Saltelli *et al.*, 2004).

The ''Extended-FAST'' method, proposed by Saltelli *et al.* (1999) builds on the previous work by Cukier *et al*. (1973) and Schaibly and Shuler (1973) and computes both first order effect S and total-effect S_T. We used the sensitivity package (Pujol, 2008) developed within the R programming environment (R Development Core Team, 2011).

RESULTS AND DISCUSSION

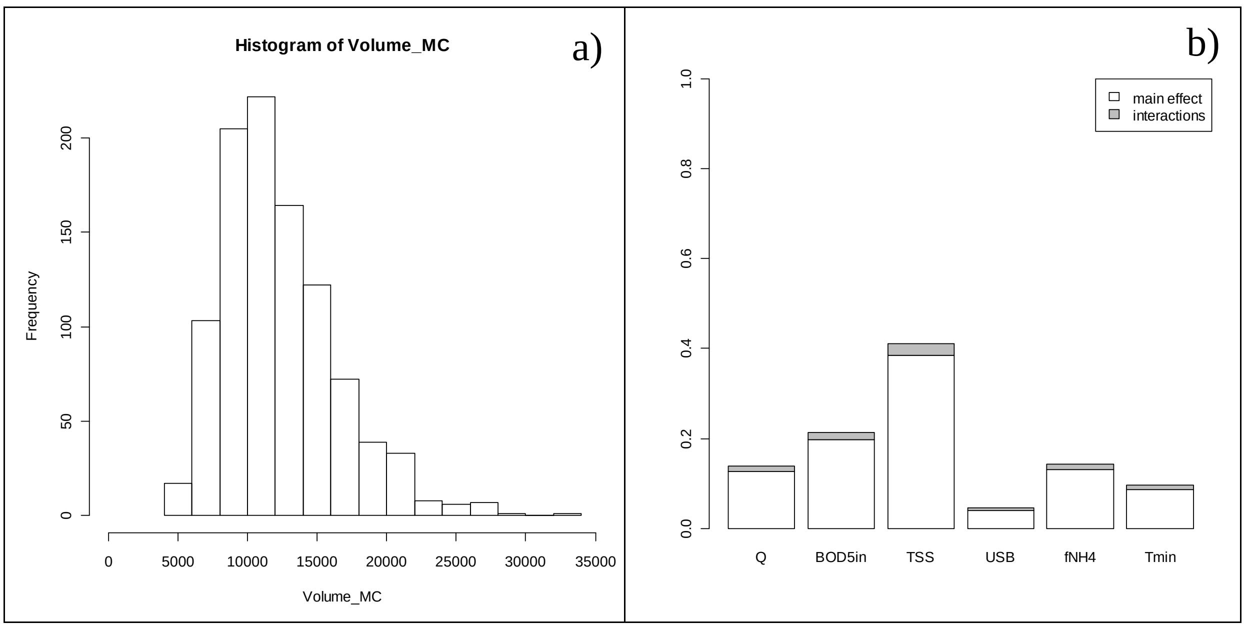

Figure 1. *Panel a) Histogram of design volumes obtained with MC simulation (mean: 12283 m^3, standard deviation: 4124 m^3). Panel b) Results from variance decomposition with 1st order – or main effects (S) and total order effects (S_T = S + interactions) for the six model factors.*

Propagating the input ranges through the design equations (MC simulation) leads to a skewed distribution of tank volumes (Figure 1 a). The design volumes range from around 5000 m^3 to 35000 m^3 and can be summarised by mean +/- standard deviation (sd) as 12300 m^3 +/- 4100 m^3.

The variance decomposition attributes the variance (sd^2) in the model output to the input factors. The main- or first order effects S are characterised by the white bars in Figure 1 b). The total effects are characterised as the sum of main effects (white) and interactions (grey): S_T = S + interactions.

For this design the sum of main effects turns out to by 0.97 which means that 97% of the variance is explained by the main- or first order effects. In this design it can be concluded that due to the small interactions the main effects and total effects turn out to be similar leading to the same conclusions for both the “factors prioritisation” and “factors fixing” setting.

Deciding on a value for the operational TSS is expected to reduce the variance of the design volume by 39%. The relative contribution of factors due to uncertainty about future load patterns and wastewater characteristics shows that uncertainty about the design load (Q (13%) and BOD_{5in} (20%)) dominates over the uncertainty of the specific sludge production USB (4%) and the minimal water temperature T_{min} (9%). The uncertainty of the owner about selecting a safety factor fNH_4 accounts for 13% of the variance in the design volume.

CONCLUSIONS

Computing first order – or main effect indices and total effect indices allows discriminating between influential and non-influential input factors to a design problem. Compared to the previously applied Standardised Regression Coefficients (SRC) no assumptions are required about the model’s behaviour.

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