Uncertainty Analysis in Environmental Systems: Monte Carlo techniques and Stochastic Differential Equations

S. Kops, P. Vanrolleghem and H. Vangheluwe
Biomath, University of Gent
Coupure Links 653
B-9000 Gent, Belgium

For the last decades there has been a growing awareness of the existence of random properties in environmental systems. One widely used technique to evaluate the random properties of environmental systems during the analysis (understanding) or design (e.g., urban drainage systems, WWTP's) of such systems is called the Monte Carlo (MC) technique. The probability density function of the modelled system is obtained by numerous simulation runs, where in each run parameter values or initial conditions are chosen from 'a priori' distributions.

Since in Monte Carlo techniques the random properties of the system are not modelled explicitly, these techniques cannot be used for control of environmental systems. Whenever random properties are to be taken into account for this purpose, this is done by adding (white) noise to the deterministic system equations. This will result in a stochastic model. When all system equations are continuous the stochastic model can be written as a stochastic differential equation (SDE) with additive noise.

Stochastic differential equations are also gaining importance in uncertainty analysis. However, additive noise, as being used in control, may yield unrealistic system variable values (such as negative concentrations). A more realistic assumption can be made by Monte Carlo techniques, namely that the parameters and initial conditions are, stochastic processes and stochastic variables, respectively. By including these stochastic processes and variables explicitly into the model one obtains SDE's with parameter noise. Moreover, by modelling the parameters as stochastic processes it is less likely to obtain unrealistic variable values. Hence, in contrast with MC techniques, a (sometimes unrealistic) constant (parameter) distribution does not have to be assumed.

In this presentation the use of Monte Carlo techniques and stochastic differential equations with parameter noise for the analysis of uncertainty in environmental systems are being compared. This will be illustrated using a model describing biodegradation in WWTP's.