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Analysis and simulation of the sludge profile dynamics in a full-scale clarifier

Jeriffa De Clercq,[1]* Martijn Devisscher,[2] Ivo Boonen,[2] Jacques Defrancq[1] and Peter A Vanrolleghem[3]

[1]*Department of Chemical Engineering and Technical Chemistry, Ghent University, Technologiepark 914, B 9052 Zwijnaarde, Belgium*
[2]*Aquafin NV, Dijkstraat 8, B 2630 Aartselaar, Belgium*
[3]*BIOMATH, Ghent University, Coupure Links 653, B 9000 Gent, Belgium*

Abstract: A one-dimensional clarifier model was assessed for its capability to describe dynamic full-scale sludge concentration profiles by using the settling properties calibrated with batch settling curve data collected by a SettloMeter®. These sludge concentration profiles and batch settling tests formed part of a detailed one-month measuring campaign on a full-scale wastewater treatment plant; the measurements showed a daily variation in settling properties. Using the settling properties obtained from batch settling tests and a one-dimensional model without dispersion, the dynamics of the full-scale clarifier were analysed and the need for dispersion clearly shown. The parameters of the dispersion model were estimated from the full-scale sludge concentration profiles. The settling properties of activated sludge can be automatically determined by fitting the model to the on-line batch settling curve measurements and are needed as input to the one-dimensional model. This model can therefore be used for operation and control. The dispersion model parameters have to be determined from dynamic sludge concentration profiles but are assumed to be constant for a specific clarifier.

Keywords: activated sludge; clarifier; dispersion; mathematical modelling; parameter estimates; settling velocity

NOTATION

A	Surface area (m^2)
C	Sludge concentration ($g\,dm^{-3}$)
D	Dispersion coefficient ($m^2\,d^{-1}$)
D_{11}, D_{22}	Dispersion parameters ($m^2\,d^{-1}$)
F	Flux ($m\,d^{-1}\,g\,dm^{-3}$)
n	Parameter of the Vesilind settling velocity function ($dm^3\,g^{-1}$)
Q	Flow rate ($m^3\,d^{-1}$)
$Q_f C_f$	Sludge load ($kg\,d^{-1}$)
s	Source term ($g\,dm^{-3}\,d^{-1}$)
SBH	Sludge blanket height (m)
SVI	Sludge volume index ($cm^3\,g^{-1}$)
t	Time (d)
V_S	Settling velocity ($m\,d^{-1}$)
V_0	Parameter of the Vesilind settling velocity function ($m\,d^{-1}$)
V_0'	Reparameterized parameter of the Vesilind settling velocity function ($m\,d^{-1}$)
z	Height (m)
α, β	Dispersion parameters (dimensionless)

Subscripts

average	Average feed
e	Effluent
f	Feed
r	Recycle
u	Underflow
1	Clarification zone
2	Thickening zone

INTRODUCTION

The activated sludge process consists of a continuously operated aeration tank and a clarifier. The clarifier produces thickened sludge for return to the aeration tank and a clarified final effluent, and is a storage tank for sludge during peak flows. Should the clarifier fail in any of these functions, suspended solids are carried over to the effluent. The performance of the clarifier determines the quality of the effluent and the efficiency of the whole purifying process.

Mathematical models attempt to represent all of the important processes in the clarifier and are of theoretical, practical and economic interest since they can be used to simulate, design and control clarifiers. One-dimensional models only describe the processes

* Correspondence to: Jeriffa De Clercq, Department of Chemical Engineering and Technical Chemistry, Ghent University, Technologiepark 914, B 9052 Zwijnaarde, Belgium
E-mail: jeriffa.declercq@UGent.be
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in the vertical dimension and are used for operation, control and operator training.[1] These models answer mass inventory questions, questions related to the recycling of activated sludge and questions about sludge blanket levels, and are the subject of this paper. To use these models in practice, they need to be validated and verified with full-scale dynamic data,[2–4] but only few such data are available in the literature.[5,6]

In the paper by De Clercq *et al*,[6] dynamic sludge concentration profile measurements are discussed and are used for the development of a new model with a height-dependent cross-sectional area and two empirical flow rate-dependent dispersion coefficients. The settling characteristics and dispersion parameters were determined from the measured sludge concentration profiles. When comparing the simulated settling velocities with those obtained from batch settling tests, the simulations overestimated the settling velocities for the lower concentrations. Although batch settling tests are commonly used to measure settling characteristics, De Clercq *et al*[6] did not use these tests to determine these characteristics. Cacossa,[7] on the other hand, obtained the settling characteristics from batch settling tests and accurately predicted experimental steady-state sludge concentration profiles. With this procedure, it is more easy to use a one-dimensional model for practical applications such as optimizing the operation of the activated sludge process since calibration of the settling parameters is achieved from simple batch settling tests.

This paper describes the dynamic full-scale sludge concentration profiles[6] with a one-dimensional model by calibrating the settling characteristics from batch settling data. First of all, it is investigated whether the settling velocity function and its parameters used by De Clercq *et al*[6] are capable of simulating the measured batch settling curves. The dispersion coefficients of De Clercq *et al*[6] are also subjected to a thorough analysis.

MATERIALS AND METHODS

Full-scale measurements

The wastewater treatment plant had a biological treatment section (with nitrification/denitrification and chemical removal of phosphorus) and a secondary clarification unit. The circular centre-fed clarifier had a diameter of 19.3 m, a sidewall depth of 1.88 m and a central depth of 2.56 m, and was operated in such a manner that the sludge blanket height lay between 0.1 and 0.5 m. The different measurements have been described by Le Poulichet[8] and De Clercq *et al*.[6] Sludge concentration profiles (Staiger-Mohilo 7210 MTS sensor), sludge bed height (Staiger-Mohilo 7210 MTS sensor), batch settling curves (SettloMeter®[9]), effluent flow rate (swedmeter LF300/T), total suspended solids concentration[10] of the inlet, recycle and effluent, and sludge volume index[10] were measured during a period of one month.

Since the sludge concentration profile and batch settling curve measurements were crucial for this investigation, their operation principle is explained. The Staiger-Mohilo 7210 MTS sensor consisted of a microprocessor-based analyser with two analogue outputs for suspended solids and height; a stepping motor and controller and a 7510 SAM sensor for optical detection of the suspended solids concentration. The SettloMeter®[9] was a down-scaled version of a secondary clarifier designed in such a way to avoid wall effects and prevent solids from bridging. With an external light source and a moving light-intensity scanner, the sludge blanket height was continuously located.

During the whole month, the feed concentration increased from 2 g dm^{-3} up to 4 g dm^{-3}, partly as a consequence of rain. For 3 days (from day 20 until 23), the recycle flow rate was halved and from day 29 until 31, the recycle flow rate was increased by 50%. All these changes resulted in marked variations in load to the clarifier (the final load was almost four times greater than initially). Figure 1 shows the measured sludge blanket height and load and Fig 2 (left) shows the measured sludge concentration profiles (from a depth of 0.5 m to 2.06 m). On 20 of the 31 days, batch settling curves were recorded with a SettloMeter®[9] and on 3 days of these 20, the recording was done for several concentrations, as shown in Table 1, giving a total of 82 settling curves (Fig 3). The SVI-values of Table 1 show that settling properties changed throughout the whole month.

Basic model description and numerical integration

The model and its numerical integration have been described by De Clercq *et al*.[6] A few important aspects of the model are given here. The continuity equation for sludge in the clarifier is the nonlinear partial differential equation:

$$\frac{\partial C(z,t)}{\partial t} = -\frac{\partial(F(C(z,t),z,t)}{\partial z} + \frac{\partial}{\partial z}\left(D(z,t)\frac{\partial C(z,t)}{\partial z}\right) + s(z,t) \quad (1)$$

where $C(z,t)$ is the sludge concentration which is dependent on height z and time t, $F(C(z,t),z,t)$ is the flux, $s(z,t)$ is the source term, $D(z,t)$ is the dispersion coefficient, which is possibly dependent on local variables and/or on input variables, such as flow rates and feed concentration $C_f(t)$. The flux $F(C(z,t),z,t)$ comprises the bulk vertical movement of water and settling. The source term $s(z,t)$ is described as a point source $Q_f(t)/A(z)C_f(t)\delta(z-z_f)$, where $A(z)$ is the surface area at a certain height z, z_f is the feed location and $\delta(z)$ is 1 for z equal to zero and zero for all other z-values. Nonlinear boundary conditions express the absence of settling and dispersion at the top and the bottom of the clarifier.

The nonlinear partial differential equation is converted to a system of ordinary differential equations by differencing the spatial derivatives of the

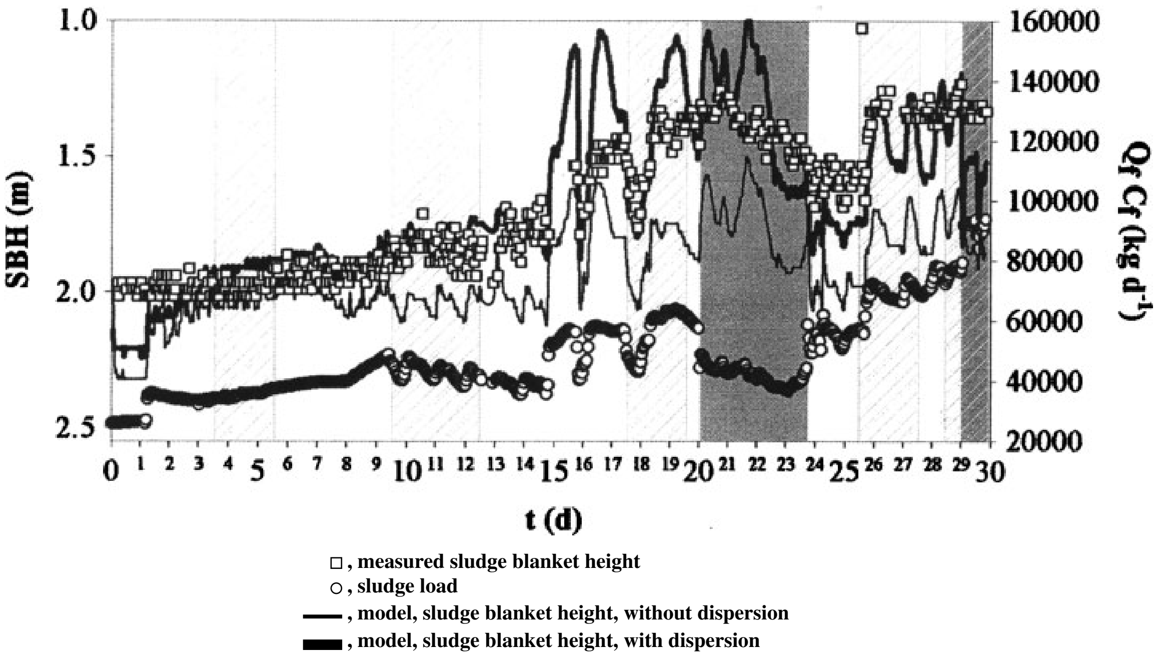

Figure 1. Simulated and measured sludge blanket height and sludge load versus time (grey area: changes of recycle flow rate;[6] hatched areas: uncertain settling properties).

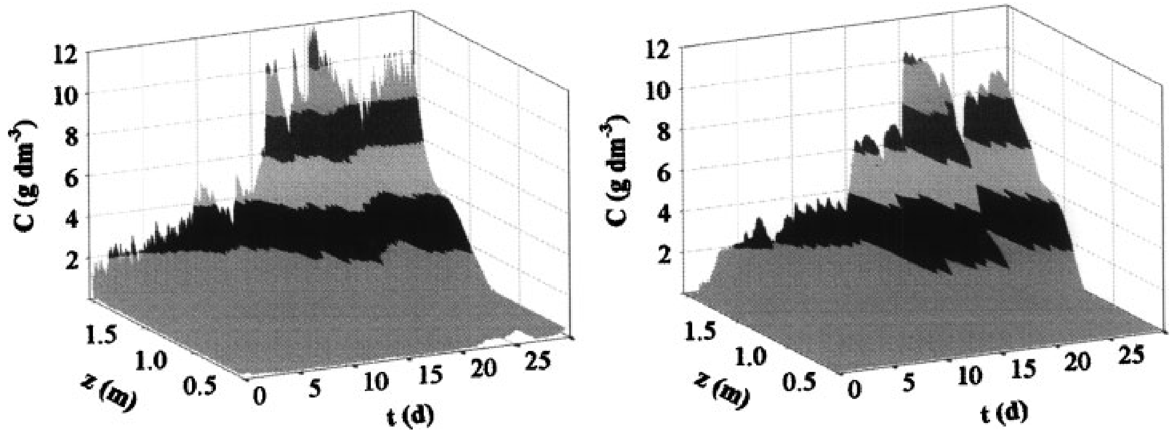

Figure 2. Measured[6] (left) and simulated (right) sludge concentration profiles (model with dispersion).

partial differential equation. The system of ordinary differential equations is integrated with LSODA.[11]

RESULTS AND DISCUSSION

Modelling batch settling curves

The settling characteristics for activated sludge are important in determining the performance of the clarifier. Batch settling experiments are an interesting information source for these characteristics as the data are only the result of the physical properties of the measuring device and the settling properties of the sludge. All the other processes besides settling, that occur in a full-scale clarifier, such as bulk flows, two- and three-dimensional dispersion, anomalies in the particulates' transport and the sludge removal procedure can be disregarded in a batch settling experiment. Batch settling is used for process monitoring, intended to measure how well sludge can be thickened in a continuous thickening process, as well as to validate thickening models. Eckenfelder and Melbinger[12] showed that the batch settling data can be transferred to the continuous sedimentation process. When a batch settling curve can be simulated/predicted, then the governing settling velocity function can be used to describe the settling behaviour of activated sludge in a clarifier.

The same model as given in the model description, but with the dispersion coefficient, source term and flow rates set to zero, is used to predict the measured batch settling curves. The flux $F(C(z,t),t)$ contains the settling velocity function $V_S(C(z,t),t)$. The simulated sludge blanket height is determined from the simulated sludge concentration profile (the model calculates sludge concentration profiles) and is taken as the height where the sludge concentration reaches $0.8\,\mathrm{g\,dm^{-3}}$ (since the lowest sludge concentration of all batch settling curves was around $0.8\,\mathrm{g\,dm^{-3}}$).

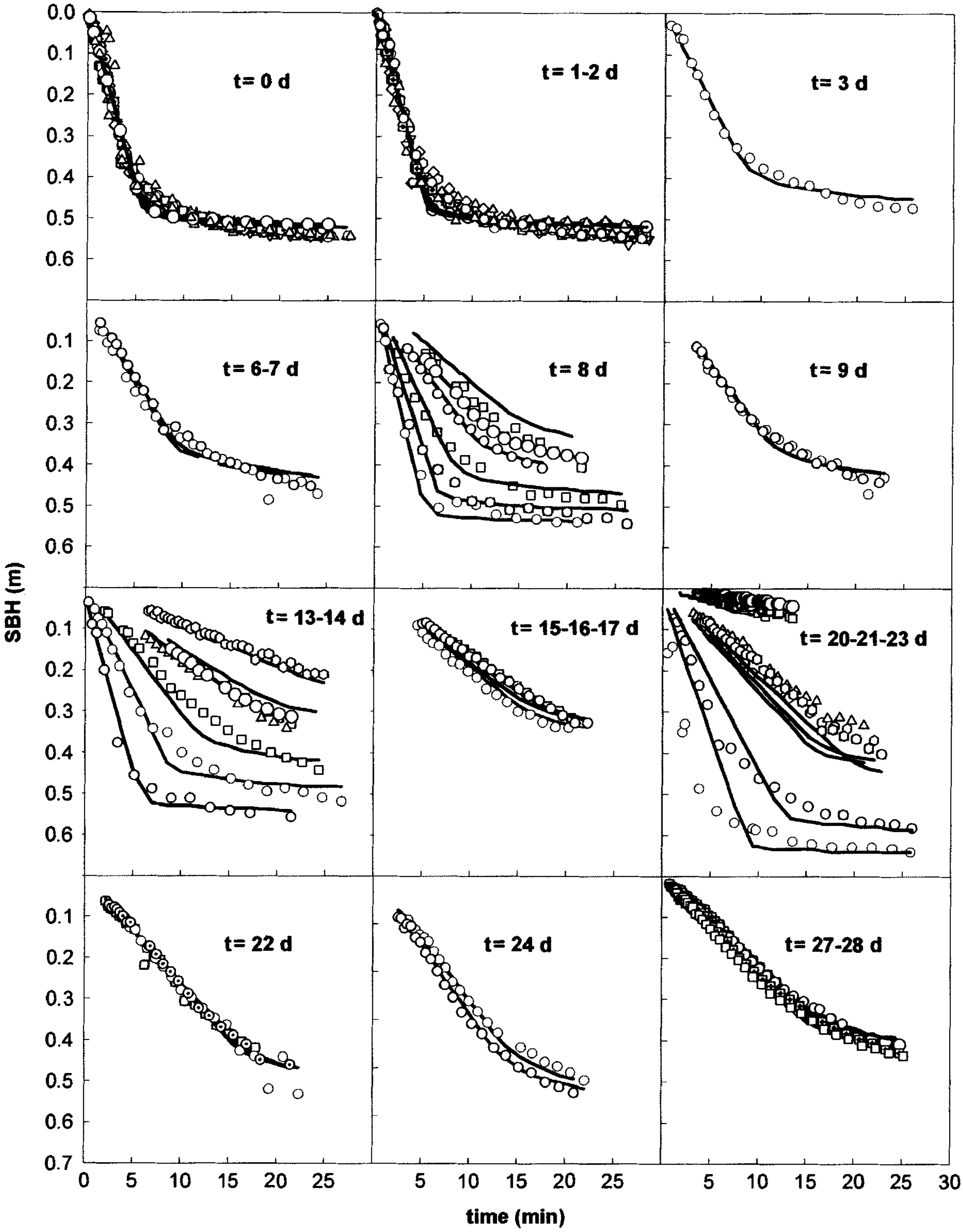

Figure 3. Predicted (line) and measured (symbols) batch settling curves, for different days as shown in each graph.

De Clercq *et al*[6] estimated the settling and dispersion parameters on the basis of the measured full-scale sludge concentration profiles and preferred the Cho[13] function over the Vesilind[14] function because of its better fit of these profiles. The Cho[13] and Vesilind[14] functions are given by, respectively:

$$V_S(C(z,t),t) = \frac{V_0(t)e^{-n(t)C(z,t)}}{C(z,t)}$$

$$V_S(C(z,t),t) = V_0(t)e^{-n(t)C(z,t)} \qquad (2)$$

In the current work, the settling parameters are estimated on the basis of the measured batch settling curves and it is investigated whether the Cho[13] function and its parameters, determined by De Clercq *et al*,[6] are capable of predicting the measured batch settling curves. Since there are 1795 data points, a parity plot is shown in Fig 4 to give an overview of

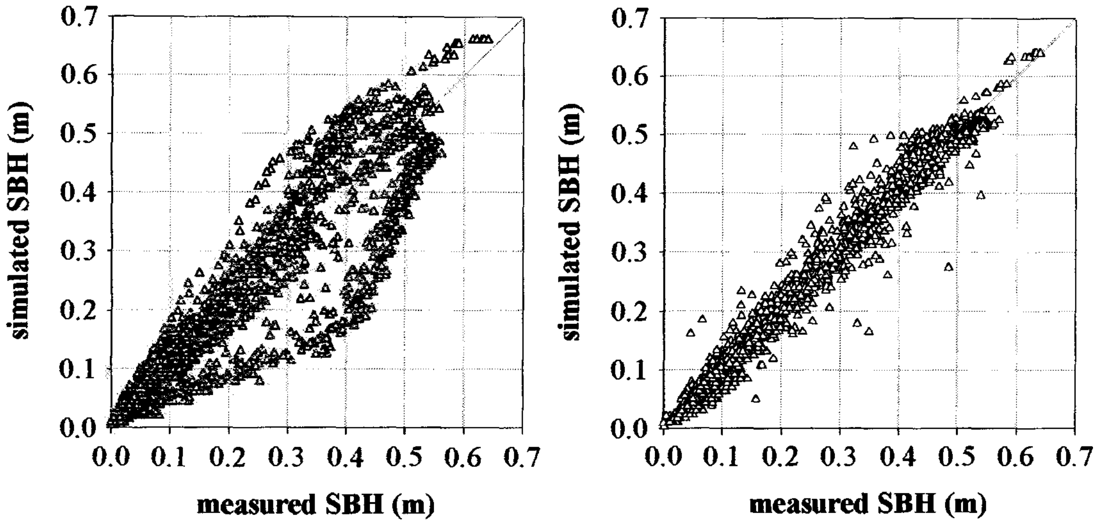

Figure 4. Predicted versus measured sludge blanket height for the Cho function (left) and for the Vesilind function (right).

Table 1. Number of batch settling curves (No) measured each day (time) at a certain sludge concentration (C) and the SVI measured off-line at the feed concentration

Time (d)	No	C (g dm^{-3})^{a}	SVI (cm^3 g^{-1})
0	23	2.1	88
1	14	2.2	91
2	2	2.1	98
3	1	2.0	121
4	—		—
5	—		—
6	1	2.3	—
7	1	2.4	—
8#	6	4.3; 3.6; 3.2; 2.4; 1.7; 1.0	162
9	2	2.9	140
10	—		155
11	—		—
12	—		—
13	2	3.3	216
14#	5	4.2; 3.5; 2.4; 1.6; 0.8	231
15	2	3.2	219
16	2	3.8	221
17	1	3.4	201
18	—		—
19	—		—
20	1	3.5	159
21	1	3.4	153
22	3	3.2	142
23#	6	9.2; 8.8; 6.6; 3.8; 2.0; 0.9	152
24	2	3.8	121
25	—		—
26	—		—
27	6	4.2	98
28	1	4.5	96
29	—		101
30	—		88

Batch settling curves are measured at different sludge concentrations, which are obtained by diluting recycle or feed sludge with effluent.

a Equals the feed concentration, except for the days marked by #.

how the Cho function is predicting the measurements. It is obvious that the agreement between calculated and measured sludge blanket heights is not good. It is therefore concluded that the model of De Clercq *et al*[6] and its parameters do not represent the correct settling properties of the activated sludge and need to be adjusted so that the batch curves are well predicted.

Since the Cho function was only used by De Clercq *et al*[6] because of its better fit of the sludge concentration profiles, and the Vesilind function is the most frequently used in the literature, the settling properties of the activated sludge, ie the batch settling curves, are modelled in this study with the Vesilind function. The parameters $V_0(t)$ and $n(t)$ of this function were estimated using the Levenberg–Marquardt algorithm.[15] The objective function for parameter estimation was the sum of squared errors between the observed and predicted sludge blanket height. Because the parameters V_0 and n are calibrated simultaneously, the exponential dependency of the settling velocity gives a correlation between both parameters. To prevent this, a reparameterization[15] is performed so that the estimated parameters V_0' and n become less correlated:

$$V_0' = V_0 \mathrm{e}^{-nC_{\mathrm{average}}} \tag{3}$$

with C_{average} the average feed concentration over the time period considered. For example, for the first day, the correlation coefficient between the parameter estimates was 0.794, while it was 0.059 with the reparameterization. The same trend is obtained for the other days (results not given).

The settling properties, ie the settling parameters, are kept constant for consequent days if the batch settling curves measured at the same concentration overlapped. The predicted and measured batch settling curves are shown in Fig 3. The agreement with the experimental data is satisfying, except for $t = 23$ days for the two fastest declining curves (these are curves for concentrations lower than C_{f}). The parity plots in Fig 4 show that the Vesilind function is predicting the measurements better than the Cho

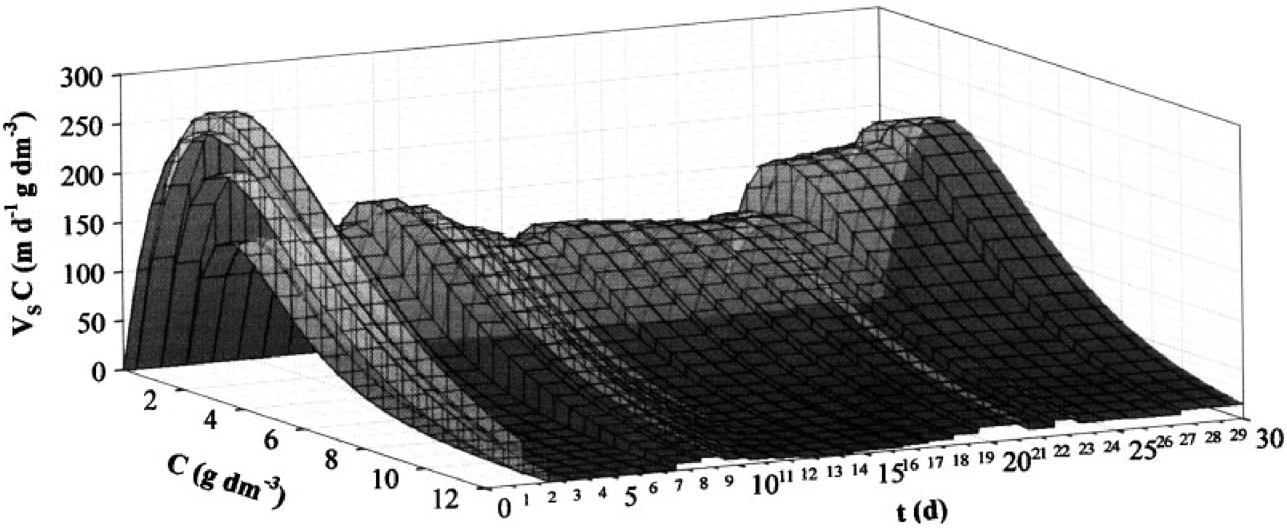

Figure 5. Settling flux versus time and concentration.

function. Figure 5 represents the varying settling flux with time.

It is illustrated here that by investigating/simulating the automatically generated settlometer batch settling curves, changing settling properties can be detected. The parameters of the settling velocity function can easily be estimated at very regular intervals. Since it is shown that the batch settling curves can be described accurately by the Vesilind function, this function and its parameters adequately describe the settling properties and could be the basis for a model that tries to simulate the behaviour of the clarifier.

Even more information about settling properties could have been gained if more batch settling curves could have been collected, ie every hour, every day. However, in this study, this was only the case for the first and second day.

Modelling sludge concentration profiles

The obtained Vesilind parameters are now input to the model to simulate the full-scale sludge concentration profiles. Since there are no batch settling curves and SVIs available at 11 of the 31 days, the settling parameters are calculated by assuming a linear variation with time. At these days (4, 5, 10, 11, 12, 18, 19, 25, 26, 29 and 30 days), the values of the settling parameters are uncertain and this has to be considered in the discussion.

Analysis/simulation of the sludge concentration profile dynamics with model without dispersion

At first, simulations with eqn (1) where dispersion is set to zero were performed with the settling properties (ie Vesilind function and its parameters) estimated from the batch settling curves, and with the measured effluent and recycle flow rate, feed concentration and clarifier's geometry.

To analyse the dynamics, the measured and simulated sludge blanket height and sludge load are shown on Fig 1. The hatched areas of Fig 1 represent the time period in which the settling properties are uncertain (no batch settling curves and SVI measurements available), so the simulations at those times cannot be compared with the measurements. The grey areas represent the time period in which the recycle flow rate was changed from its normal value (first period: decrease of recycle flow rate; second period: increase of recycle flow rate).

At the beginning, the measured sludge blanket height seems too high in comparison with the simulations but in 50% of the measurements in this period, the sludge blanket height was somewhere below 2.06 m (which is below the detection level of the sensor) and is therefore not shown in Fig 1. Taking this into consideration, it can be concluded that the simulated sludge blanket height corresponded well with the measurements until day 9.

From that day on, the sludge blanket height was underpredicted (except around day 15–16) but still showed the same trend as the measurements. From day 9 on, the settling properties deteriorated until day 20 and improved afterwards. This is reflected in both the simulated and the measured sludge blanket height.

Besides the changing settling properties, the load also changed during the investigated time period. Simulations and measurements both showed an increasing sludge blanket height for increasing load and vice versa. Changing the recycle flow rate gave the expected change in sludge blanket height for the simulations and the measurements. Those changes were more pronounced, though, for the simulations.

To conclude, in the investigated time-period, settling properties, flow rates and feed concentration changed with time. The varying settling properties were taken into account by providing the varying settling parameters obtained from the SettloMeter®[9] data. The varying flow rates and feed concentrations are input to the model, so they are considered too. However, this is not sufficient to get a good prediction of the sludge blanket height, since there are other processes occurring besides settling and advection. Dispersion should account for these processes.

Analysis/simulation of the sludge concentration profile dynamics with model with dispersion

It is attempted to model this dispersion with the two flow rate-dependent dispersion coefficients of De Clercq *et al*[6]:

$$D_1(t) = D_{11} e^{\alpha \frac{Q_e(t)}{Q_f(t)}} \qquad D_2(t) = D_{22} e^{\beta \frac{Q_u(t)}{Q_f(t)}} \tag{4}$$

with $D_1(t)$ and $D_2(t)$ the dispersion coefficients in the clarification zone ($z < z_f$) and thickening zone ($z \geq z_f$) respectively, D_{11}, α, D_{22} and β the dispersion parameters, $Q_e(t)$ the effluent flow rate, $Q_f(t)$ the feed flow rate and $Q_u(t)$ the underflow rate. The estimation of the dispersion parameters is again done with the Levenberg–Marquardt algorithm.[15] The objective function for parameter estimation was the sum of squared errors between the observed and predicted sludge concentration profiles (33 300 data points, ie 550 profiles each with 60 measurements).

The residual sum of squared errors was more than halved by adding dispersion to the model. This gives a better fit of the sludge concentration profiles. The measured and predicted sludge concentration profiles are compared in Fig 2. The predicted profiles act more nervously but the trends are well approximated/simulated.

Figure 1 gives the measured and predicted sludge blanket heights. The simulated sludge blanket height was, of course, greater than the simulations with the model without dispersion. The simulations gave a fairly good prediction of the measurements but showed more correlation with the load than the measurements indicate. The dispersion in the clarification zone (maximum $\pm 8.4\,m^2\,d^{-1}$) was higher than in the thickening zone (maximum $\pm 0.15\,m^2\,d^{-1}$). The higher dispersion in the upper zone could be attributed to the flow pattern in the clarifier which can exhibit several recirculation zones, mainly located in the clarification zone.[16] The α and β parameters in the dispersion functions were 0.236 and -0.741 respectively.

The model adequately describes the settling characteristics which are obtained by estimating the settling velocity function parameters on the basis of measured batch settling curves. Dispersion was added to the model to account for its non-ideal flow behaviour. The one-dimensional model reasonably predicts the profile dynamics and may be used for practical applications such as optimizing the operation of the activated sludge process.

The settling characteristics were found by fitting the model to the on-line measured batch settling curves. In the current case, these curves were measured with a SettloMeter®,[9] but this could be done with any device capable of measuring batch settling curves, which only reflect the settling behaviour (ie are not dependent on the measurement device itself). Every measured batch settling curve was used as input to the one-dimensional model through the estimated settling velocity function parameters. Measuring a batch settling curve took about 40 min, simulating/estimating about 1 min. The current settling properties were subsequently used as input for the one-dimensional model (next to flow rates and feed concentration) that simulates/predicts the expected sludge concentration profiles. For each specific clarifier, the dispersion parameters (relating the dispersion coefficient to the flow rates) need to be estimated once from profile measurements.

CONCLUSIONS

A one-dimensional clarifier model is shown in this work to be capable of describing the dynamics of the full-scale sludge concentration profiles[6] after calibration of the settling properties with on-line collected batch settling curves. The one-dimensional clarifier model has a height-dependent cross-sectional area, two flow rate-dependent dispersion coefficients and the Vesilind settling velocity function.

The same model, but without dispersion and bulk flow, is used to determine the settling properties by calibrating the Vesilind parameters to fit the measured batch settling curves. Since it is shown that the batch settling curves can be described accurately by the Vesilind function, this function and its parameters adequately describe the settling properties and should be the basis for each model that tries to simulate the behaviour of the clarifier. With these settling properties, the full-scale sludge concentration profile measurements are analysed and the need for dispersion is clearly shown. The parameters of the dispersion model[6] are estimated from the full-scale sludge concentration profiles.

The settling properties can change on a daily basis as shown in the measurements and have to be determined daily, preferably on-line, for example with the SettloMeter®.[9] The Vesilind parameters can then be automatically determined from modelling the on-line batch settling curve measurements and the one-dimensional model can be used for operation and control. The dispersion parameters have to be determined from dynamic sludge concentration profiles but can be assumed to be constant for a specific clarifier.

With independent measurements at another wastewater treatment plant, the functional form of the dispersion coefficients needs to be validated before the model can be used for operation and control. The SettloMeter® can be equipped with an automatic dilution device so that settling curves can be measured at different concentrations.

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