WATER RESEARCH 42 (2008) 781–791

Available at www.sciencedirect.com

ScienceDirect

journal homepage: www.elsevier.com/locate/watres

# Extending and calibrating a mechanistic hindered and compression settling model for activated sludge using in-depth batch experiments

*Jeriffa De Clercq*[a,c,*], *Ingmar Nopens*[b], *Jacques Defrancq*[c], *Peter A. Vanrolleghem*[b,d]

[a]Faculty of Applied Engineering Sciences, University College Ghent, Schoonmeersstraat 52, 9000 Gent, Belgium
[b]BIOMATH: Department of Applied Mathematics, Biometrics and Process Control, Ghent University, Coupure Links 653, 9000 Gent, Belgium
[c]Department of Chemical Engineering and Technical Chemistry, Ghent University, Technologiepark 914, 9052 Zwijnaarde, Belgium
[d]ModelEAU Département de génie civil, Pavillon Pouliot, Université Laval Québec, G1K 7P4, QC, Canada

ARTICLE INFO

*Article history:*
Received 22 March 2007
Received in revised form
15 August 2007
Accepted 15 August 2007
Available online 14 September 2007

*Keywords:*
Mechanistic model
Sedimentation
Activated sludge
Hindered settling
Compression
Inverse modelling

ABSTRACT

Currently, no mechanistic model is available in wastewater industry that can accurately describe the batch settling behaviour of activated sludge. Such a model, which is based on the fundamental mass and force balances for water and solids, is extended and applied in this work and excellently describes batch settling experiments for sludges originating from two different wastewater treatment plants.

The mechanistic model contains a Kynch batch density function $f_{bk}$ (hindered settling) and an effective solids stress function $\sigma_e$ (compression). Initial settling velocities were obtained from detailed spatio-temporal dynamic solids concentration profiles measured with the aid of a radiotracer [De Clercq, J., Jacobs, J., Kinnear, D.J., Nopens, I., Dierckx, R.A., Defrancq, J., Vanrolleghem, P.A., 2005. Detailed spatio-temporal solids concentration profiling during batch settling of activated sludge using a radiotracer. Water Res. 39(10), 2125–2135]. Moreover, inverse modelling calculations were performed using the same data set. Both calculations showed that (1) the power function of Cole gave acceptable results and (2) a single effective solids stress function could be found when a time-dependent compression solids concentration $C_C$ was considered. This compression solids concentration is found just below the sludge blanket and is readily calculated from the solids concentration profiles. Given these time-evolutions, the effective solids stress values exhibit a uniform logarithmic relationship with the difference between the solids concentration and the compression solids concentration.

The descriptive power of the model indicates a good potential for wider applicability of the model.



## 1. Introduction

The effectiveness of the activated sludge process is highly dependent on the settling characteristics of the mixed liquor. The influent wastewater composition and the operating conditions of the biological tanks influence the composition of the microbial floc and hence the settling characteristics. Knowledge of the settling characteristics of the mixed liquor is essential for the proper design and operation of clarifiers (Jin et al., 2003; Mines et al., 2001). Those characteristics are

*Corresponding author.
E-mail address: Jeriffa.DeClercq@hogent.be (J. De Clercq).
0043-1354/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.watres.2007.08.040

**Nomenclature**

| | |
|---|---|
| $a$ | settling parameter |
| $A$ | cross sectional area, $L^2$ |
| $b$ | settling parameter |
| $C$ | solids concentration, $ML^{-3}$ |
| $C_C$ | compression solids concentration, $ML^{-3}$ |
| $C_{min}$ | non-settleable solids concentration, $ML^{-3}$ |
| $C_0$ | initial solids concentration, $ML^{-3}$ |
| $f_{bk}$ | Kynch batch density function, $ML^{-2}T^{-1}$ |
| $g$ | gravity constant, $LT^{-2}$ |
| $H$ | total depth, L |
| $J$ | objective function |
| $n$ | Vesilind (1968) settling parameter, $L^3M^{-1}$ |
| $N$ | number of observations |
| $r_h$ | Takács settling parameter, $L^3M^{-1}$ |
| $r_p$ | Takács settling parameter, $L^3M^{-1}$ |
| $t$ | time, T |
| $V_{hindered}$ | hindered settling velocity, $LT^{-1}$ |
| $V_0$ | Vesilind (1968) settling parameter, $LT^{-1}$ |
| $V_{0max}$ | Takács settling parameter, $LT^{-1}$ |
| $V_S$ | settling velocity, $LT^{-1}$ |
| $y$ | observation |
| $\hat{y}_i$ | model prediction |
| $z$ | depth in settling column, L |

*Greek letters*

| | |
|---|---|
| $\alpha$ | effective solids stress parameter, $MT^{-2}L^{-1}$ |
| $\beta$ | effective solids stress parameter, $ML^{-3}$ |
| $\rho_s$ | solids mass density, $ML^{-3}$ |
| $\sigma_e$ | effective solids stress, $MT^{-2}L^{-1}$ |
| $\theta$ | parameter set |

commonly measured using batch settling tests. Currently, however, no (mechanistic) model is available in wastewater industry that can accurately describe the entire batch settling behaviour of activated sludge, measured using these batch settling tests. Most models are empirical (Vesilind, 1968; Takács et al., 1991; Dupont and Henze, 1992; Härtel and Pöpel, 1992; Otterpohl and Freund, 1992; Cho et al., 1993) and describe either (1) solely hindered settling, (2) hindered settling and compression settling and (3) hindered settling and settling at low solids concentrations. The Vesilind and Takács functions are the most frequently used. They are, respectively, given by

$$V_S = V_0 e^{-nC}, \quad (1)$$

$$V_S = \min(V_{0max}, V_0(e^{-r_h(C-C_{min})} - e^{-r_p(C-C_{min})})). \quad (2)$$

However, these empirical models have no relationship with the physical properties of activated sludge flocs and solid–water interaction. Kinnear (2002) was one of the few who recognized this and considered the use of densities, viscosity,... to model the batch settling behaviour. Unfortunately, his mechanistic model was not capable of predicting a complete batch curve (i.e. over the full concentration range). Hence, there is still a need to describe the batch settling behaviour of activated sludge in a more fundamental and scientifically sound way.

In other application areas (e.g. flocculated slurries), more fundamental research has been performed on the modelling of batch settling of flocculated suspensions. The main difference between those mechanistic models is the use of different expressions for the hindered settling flux, i.e. the Kynch batch density function $f_{bk}$, and the effective solids stress $\sigma_e$ (e.g. Shirato et al., 1970; Shih et al., 1986; Font, 1991; Bergström, 1992; Holdich and Butt, 1997; Diplas and Papanicolaou, 1997; Zheng and Bagley, 1998, 1999; Karl and Wells, 1999; Bürger et al., 2000a). Whereas the Kynch batch density functions $f_{bk}$ have been described by a variety of equations, most models have used power or exponential laws to describe the effective solids stress. A detailed review of the cited models can be found in De Clercq (2006).

A mechanistic settling model is based on the conservation of mass and momentum of both water and solids. The forces acting on the solids are gravity, buoyancy, liquid pressure, friction and effective solids stress. The model for batch settling of activated sludge, after making some assumptions and introducing the Kynch batch density function $f_{bk}(C)$ and the effective solids stress $\sigma_e(C)$ (Bustos et al., 1999; De Clercq, 2006), is an initial-boundary value problem of a second-order partial differential equation describing the solids concentration as a function of depth and time

$$\frac{\partial C}{\partial t} = -\frac{\partial f_{bk}}{\partial z} + \frac{\partial}{\partial z}\left(f_{bk}(C)\frac{\rho_s}{\Delta\rho g C}\frac{d\sigma_e}{dC}\frac{\partial C}{\partial z}\right) \quad (3)$$

with simple kinematic zero-flux boundary and initial conditions

$$f_{bk}\left(1 - \frac{\rho_s}{\Delta\rho g C}\frac{d\sigma_e}{dC}\frac{\partial C}{\partial z}\right)_{z=0,z=H} = 0, \quad (4)$$

$$C_{z,0} = C_0. \quad (5)$$

When $C \leqslant C_C$, with $C_C$ the compression solids concentration (i.e. the concentration above which compression occurs and an effective solids stress exists), the second term on the RHS of (3) is zero and only hindered settling (first term) exists. When $C > C_C$, an effective solids stress exists and the settling is comprised of a hindered settling term (first term on the RHS of (3)) and a term due to the effective solids stress (compression, second term on the RHS of (3)).

In this paper the detailed batch experimental data of De Clercq et al. (2005) are used to determine the most appropriate functional relationship for the Kynch batch density function and the effective solids stress and to estimate the parameters of the respective functions. The model should be able to describe the following phenomena (De Clercq et al., 2005):

- hindered and compression settling;
- hindered settling rates decrease with increasing initial solids concentrations;
- the compression solids concentration $C_C$ is time-dependent.

The parameters of the Kynch batch density function and the effective solids stress function are obtained from observed initial settling velocities and inverse modelling in which all solids concentration profiles and not only the ones at equilibrium are considered.

## 2. Material and methods

### 2.1. Batch settling experiments

In De Clercq et al. (2005), novel measurement techniques were developed which allow to measure the time-evolution of solids concentration profiles with sufficient accuracy during the batch settling of activated sludge.

The radiotracer Tc-99m Sestamibi and 2 gamma cameras were used to obtain high time and spatial resolution solids concentration profiles during the batch settling of activated sludge in a pilot-scale column with a height of 1 m and an inner diameter of 386 mm, large enough to avoid wall effects. Sludge was collected from two different municipal wastewater treatment plants close to Ghent (Belgium): Destelbergen and Deinze. Dynamic solids concentration profiles with three different initial concentrations were obtained for each sludge. The pilot-scale dynamic solids concentration profiles not only showed hindered settling but (i) the equilibrium concentration profiles and (ii) the iso-concentration lines clearly showed that compression is taking place:

(i) the equilibrium profiles (no change in profiles for a period of more than 20 min) exhibited an increasing concentration towards the bottom (Fig. 1 left), while a suspension undergoing only hindered settling has an equilibrium profile with a constant concentration for which the settling velocity is zero;
(ii) when only hindered settling is occurring, the iso-concentration lines are straight lines and propagate from the origin; when compression occurs too, the iso-concentration lines become curved and arise from the bottom of the column at different values of time (Concha and Bürger, 2002; Font and Laveda, 2000; Fig. 2 shows this for one of the six experiments).

Equilibrium compression solids concentrations can be estimated from the equilibrium profiles (Fig. 1 right) by setting (3) equal to zero:

$$\sigma_e(C_{\text{equilibrium}}) = \frac{\Delta\rho g}{\rho_s}\int_0^z C_{\text{equilibrium}}\,dz \tag{6}$$

and are between 7 and 10 g/l depending on the origin of the sludge and the initial solids concentration. The iso-concentration lines showed that the compression solids concentration $C_C$ at the beginning should be around the initial solids concentration. Those observations (initial $C_C$ around $C_0$ and equilibrium $C_C$ around 7–10 g/l) resulted in a

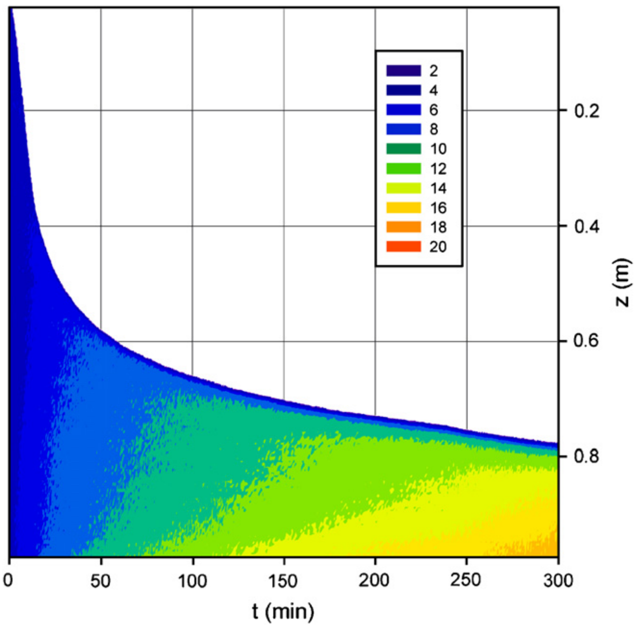


**Fig. 2 – Iso-concentration contour plots (different concentrations shown in legend) during batch settling of Destelbergen sludge ($C_0 = 3.23$ g/l).**

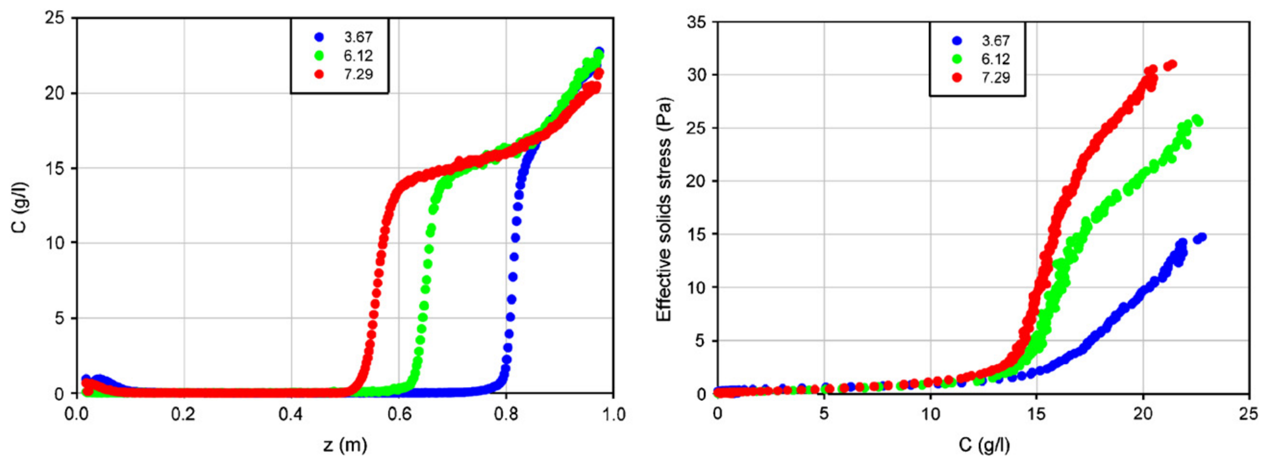


**Fig. 1 – Equilibrium solids concentration profiles (left) and effective solids stress versus solids concentration (right) for different initial concentrations (shown in legend) for the Deinze sludge (De Clercq et al., 2005).**

**Table 1 – Solids concentrations, corresponding initial settling velocities and solids density of the batch settling experiments of De Clercq et al. (2005)**

| Sludge origin | $C_0$ (g/l) | $V_{\text{hindered}}$ (m/d) | $\rho_s$ (kg/m$^3$) |
|---|---|---|---|
| Destelbergen | 2.40 | 69.18 | 1762 ± 19 |
| | 3.23 | 44.36 | 1753 ± 36 |
| | 4.30 | 24.67 | 1714 ± 6 |
| Deinze | 3.67 | 82.93 | 1943 ± 42 |
| | 6.12 | 24.45 | 1898 ± 57 |
| | 7.29 | 15.28 | 1881 ± 27 |

time-dependent compression solids concentration as modelled by Diplas and Papanicolaou (1997) and suggested by Kinnear (2002).

The batch settling curves of De Clercq et al. (2005) showed an induction period (recovery from initial disturbances such as mixing) which is omitted from the experimental data as described in De Clercq (2006), keeping in mind that the slope of the curve becomes steeper during the induction period. The initial settling velocities were calculated from the gradient of the resulting batch settling curve and are shown together with the measured solids density (pyknometer method, ISO/DTS 17892-3-2003) in Table 1.

## 3. Numerical integration

The second-order parabolic model (3) simplifies into a first-order hyperbolic type if the solids concentration is less than the compression solids concentration ($C \leqslant C_C$) as the second term of the RHS of the equation vanishes. The model is thus in fact a nonlinear mixed hyperbolic-parabolic partial differential equation. The first-order spatial term (i.e. the Kynch batch density function) and the boundary conditions of the model are nonlinear. It is well known that nonlinear hyperbolic equations give rise to discontinuities (LeVeque, 1992). An example of such a discontinuity during settling is the suspension-liquid interface.

Due to this nonlinear, mixed hyperbolic–parabolic PDE, solutions are discontinuous and conservative methods are needed to integrate the model (Bürger et al., 2000b). Upwind differencing is such a conservative discretisation method and is used for the first-order spatial terms: it stabilizes profiles which are liable to undergo sudden changes, such as discontinuities and other large gradient profiles (Bürger et al., 2000b). Since the Kynch batch density function is a non-monotone function, the generalized upwind flux of Engquist and Osher (1981) is used for this term (Evje and Karlsen, 2000; Bürger and Karlsen, 2001; Bürger et al., 2004). Conservative discretisation of the second-order spatial term is done with central differencing. The number of discretisation points (layers) is a parameter of the numerical integration. From the authors' experience, 200 layers are a suitable trade-off between convergence and calculation time. To ensure convergence of the resulting scheme to the physically relevant solution of the model, the following stability condition must be satisfied (Bürger et al., 2004):

$$\frac{1}{A}\left(A\max_C\left|f'_{bk}\right|\frac{\Delta t}{\Delta z}+2\max_C\left|f_{bk}\frac{\rho_s}{\Delta\rho g C}\frac{\partial\sigma_e}{\partial C}\right|\frac{\Delta t}{(\Delta z)^2}\right)\leqslant 1. \quad (7)$$

The spatial discretisation of the model equations gives a system of 200 first-order ordinary differential equations. The temporal concentration gradient is subsequently integrated and the time-step $\Delta t$ is determined by setting the LHS of (7) equal to 0.98.

## 4. Estimation of the model parameters

Parameters were estimated using the Levenberg–Marquardt algorithm (Marquardt, 1963). The objective function for parameter estimation, $J$, which has to be minimized, is the sum of squared errors (SSE) function:

$$J(\theta)=\sum_{i=1}^{N}(y_i-\widehat{y_i(\theta)})^2. \quad (8)$$

The measurement errors were checked and found to be Gaussian, uncorrelated and showed a constant variance which allows least squares estimation (Dochain and Vanrolleghem, 2001). The parameter vector $\theta$ contains the parameters of the Kynch batch density function and/or the effective solids stress function.

## 5. Inverse modelling

Integrating the concentration profiles at a given time instant with respect to height allows to calculate the heights above which 1%, 2%, 3%, ..., 99% of the total mass of solids is located (Tiller et al., 1991; Been and Sills, 1981; Bürger et al., 2001). The succession of these heights with respect to time yields curves that may be considered as isomass lines of solids separated by 1% from the remaining 99% (and so on) of total mass of the

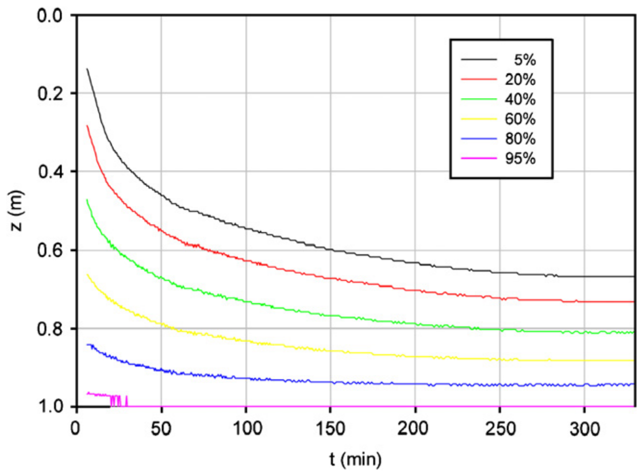


**Fig. 3 – Isomass (% of initial mass above) lines calculated from the measured concentration profiles during the batch settling of the Deinze sludge ($C_0 = 6.12$ g/l).**

sludge. These isomass lines were calculated for all six batch settling experiments (see Fig. 3 for one of them).

By definition the observed and net (combination of downward hindered settling and upward compression) settling velocity, $V_S$, is simply the gradient of the isomass line at a specific time. For each isomass line, at each time, this settling velocity is calculated. At each point of an isomass line, the solids concentration is known and with a known Kynch batch density function, the calculated settling velocity can be used to obtain values of the effective solids stress $\sigma_e$ (Eq. (10)) through the following expression (deduced from (3)) which consists of the downward hindered settling and the upward compression settling:

$$V_S = \frac{f_{bk}}{C}\left(1 - \frac{\rho_s}{\Delta\rho g C}\frac{d\sigma_e}{dC}\frac{\partial C}{\partial z}\right). \quad (9)$$

## 6. Results and discussion

### 6.1. Kynch batch density function $f_{bk}$ (hindered settling)

Different Kynch batch density functions $f_{bk}$ are reported in literature (e.g. Shirato et al., 1970; Shih et al., 1986; Font, 1991; Bergström, 1992; Holdich and Butt, 1997; Diplas and Papanicolaou, 1997; Zheng and Bagley, 1998, 1999; Karl and Wells, 1999; Bürger et al., 2000a). Observed initial settling velocities were used to find parameter estimates of the different Kynch batch density functions. The Kynch batch density function values were calculated from the observed initial settling velocities and the corresponding solids concentrations. The objective function for parameter estimation was the sum of squared errors (8) between the observed and predicted Kynch batch density values.

From this analysis (more details can be found De Clercq, 2006), the well-known Vesilind function (Vesilind, 1968) was found to be significantly better than eight other functions tested as it gave acceptable results for a concentration range from 0 to 25 g/l (positive function values) and satisfied most of the conditions of Kynch (1952).

Next, inverse modelling using the measured solids concentration profiles was applied to determine whether the Vesilind function (determined on the basis of observed initial settling velocities and corresponding solids concentrations) is indeed the appropriate function to model hindered settling. Since there is noise on the solids concentration measurement, the calculation of the concentration gradient in (9) is not straightforward. Moreover, the effective solids stress function $\sigma_e$ is of interest and not its gradient. Hence, to resolve these two issues, (9) was numerically integrated to

$$\sigma_e = \sum_{z=0}^{z=H}\left(1 - \frac{V_S}{f_{bk}/C}\right)\frac{\Delta\rho g C}{\rho_s}\Delta z. \quad (10)$$

The results of these calculations when using the Vesilind function (see De Clercq, 2006) showed that there is no single effective solids stress function $\sigma_e(C)$ which is able to describe all data points calculated with (10), even with a compression solids concentration $C_C$ (i.e. for which $\sigma_e = 0$) that for example increases with time. Comparison of the calculated effective solids stress (10) with the effective solids stress calculated from the equilibrium solids concentration profiles (Fig. 1 right) also showed that the model with the Vesilind function gave too low effective solids stress values. It was concluded that the Vesilind function is incorrect when trying to model complete batch settling solids profiles (De Clercq, 2006). Moreover, it was deduced that to obtain a single effective solids stress function, with a time-dependent compression solids concentration $C_C$, the settling velocity at concentrations higher than the initial concentrations (i.e. when the sludge becomes thickened during settling) should be higher than the velocities predicted by the Vesilind function. The power function of Cole (1968), evaluated by Cho et al. (1993) and Grijspeerdt et al. (1995), yields higher settling velocities for higher concentrations and, hence, could serve as alternative:

$$f_{bk} = aC^{-b}. \quad (11)$$

However, this function gives an infinite $f_{bk}$ for a zero solids concentration and does not have a maximum. This can be resolved either by imposing a maximum settling velocity or by using another function for the lower solids concentrations: here, a maximum settling velocity of 250 m/d was imposed. To evaluate how this power function (with parameters estimated from the initial settling velocity data) affects the effective solids stress function, inverse modelling calculations are shown in Fig. 4. These calculations gave good agreement with the equilibrium data and showed a single effective solids stress function when a time-dependent compression solids concentration $C_C$ is considered (corresponding to a shift of the $\sigma_e$-curves of Fig. 4 at different time instants to higher concentrations). It can be concluded that the Cole function is performing better than the Vesilind function. It was therefore retained as the Kynch batch density function.

The time-dependent compression solids concentration $C_C$, i.e. the concentration above which an effective solids stress exists, is located at the sludge blanket height, as was already shown in De Clercq et al. (2005). Before determining an appropriate functional relationship for the effective solids stress, the evolution of the compression solids concentration $C_C$ needs to be determined from the solids concentration profiles.

### 6.2. Evolution of the compression solids concentration $C_C$

The time-dependent compression solids concentration $C_C$ was calculated from the solids concentration profiles in the following way. At the sludge blanket height, a discontinuity exists which in reality boils down to a large concentration gradient around the initial solids concentration. Just below the sludge blanket height, the profile is much smoother (i.e. it has smaller concentration gradients). When the concentration just at the sludge blanket height would be considered as the compression solids concentration, this compression solids concentration would remain at the initial solids concentration value, i.e. it would not be time-dependent, which is contradictory to the findings. Therefore, it is suggested that the compression solids concentration $C_C$ is located just below the discontinuity of the sludge blanket height, where the concentration gradients are stabilized. It is proposed here to define the compression solids concentration $C_C$ to be the concentration at which the concentration gradient reaches values below 200 g/l/m within the sludge blanket. The critical value of 200 g/l/m was determined from calculations of the concentration gradient of the measured solids concentration profiles. With these time-dependent compression solids concentrations, an effective solids stress functional relationship can be deduced.

### 6.3. *Effective solids stress function $\sigma_e$ (compression)*

For each experiment, the effective solids stress was plotted versus the difference between the solids concentration and the compression solids concentration $C_C$ (see De Clercq, 2006). Especially the experiments at higher initial solids

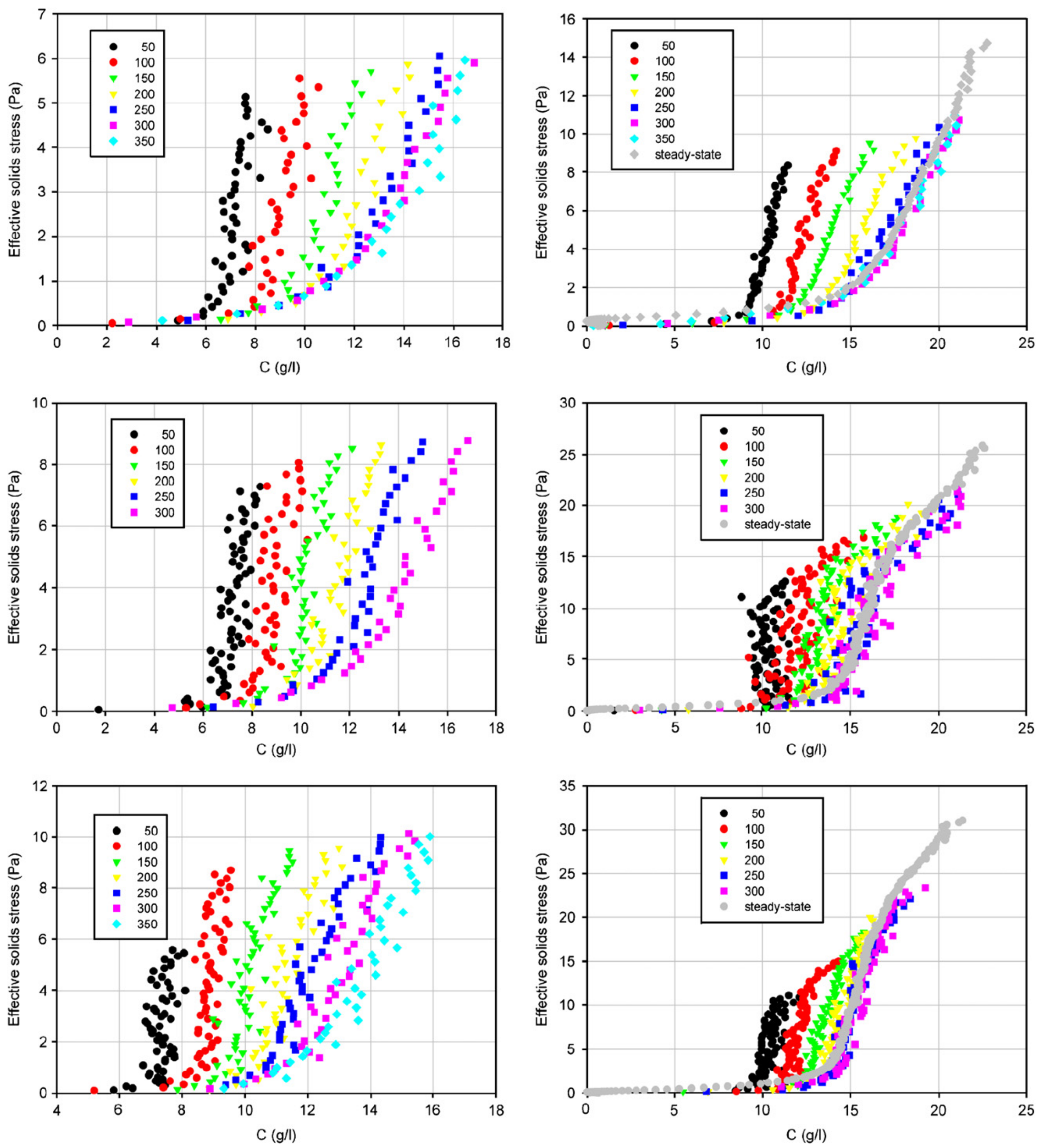


**Fig. 4 – Calculated effective solids stress versus solids concentration at different time instants (indicated in minutes in legend) during batch settling of Destelbergen sludge (left; top: $C_0 = 2.40$ g/l; middle: $C_0 = 3.23$ g/l; bottom: $C_0 = 4.30$ g/l) and Deinze sludge (right; top: $C_0 = 3.67$ g/l; middle: $C_0 = 6.12$ g/l; bottom: $C_0 = 7.29$ g/l); calculations are performed with the power function of Cole (1968), except for the curves at steady-state (grey symbols, De Clercq et al., 2005).**

concentrations showed that the most frequently used power or exponential functions were not capable of describing the calculated effective solids stresses since those functions have an increasing gradient for higher concentrations, which is opposite to the data shown in Fig. 4 (most clearly seen for the experiment on Deinze sludge with initial concentration of 6.12 g/l).

Hence, the following logarithmic function with two parameters $\alpha$ and $\beta$ was fitted to the calculated effective solids

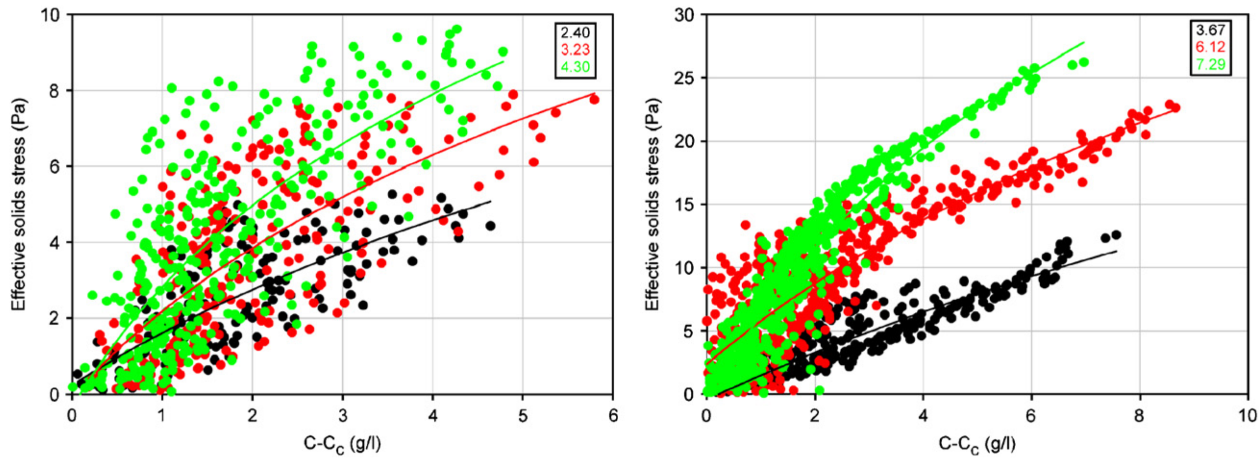


**Fig. 5 – Logarithmic effective solids stress function (line) and calculated effective solids stress (symbol) versus $C - C_C(t)$ during batch settling (left: Destelbergen; right: Deinze) (different initial solids concentrations are given in g/l in legend).**

**Table 2 – Final parameter estimates of the power batch density function (Cole, 1968) (parameters a and b) and logarithmic effective solids stress function (parameters $\alpha$ and $\beta$) for the batch settling experiments of De Clercq et al. (2005)**

| Destelbergen sludge | | $C_0 = 2.40$ g/l | $C_0 = 3.23$ g/l | $C_0 = 4.30$ g/l |
|---|---|---|---|---|
| a $(m/d(g/l)^{b+1})$ | 433 | | | |
| b (–) | 0.94 | | | |
| $\alpha$ (Pa) | 7.00 | | | |
| $\beta$ (g/l) | | 2.90 | 1.76 | 1.17 |
| Deinze sludge | | $C_0 = 3.67$ g/l | $C_0 = 6.12$ g/l | $C_0 = 7.29$ g/l |
| a $(m/d(g/l)^{b+1})$ | 3588 | | | |
| b (–) | 1.70 | | | |
| $\alpha$ (Pa) | 18.24 | | | |
| $\beta$ (g/l) | | 8.27 | 2.60 | 2.12 |

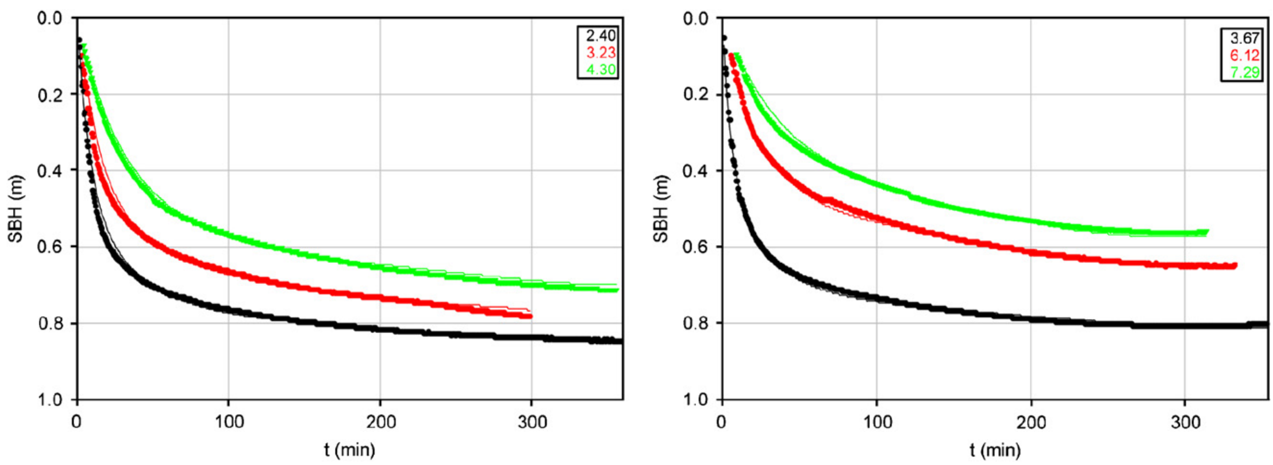


**Fig. 6 – Simulated (lines) and experimental (symbols, De Clercq et al., 2005) batch settling curves (left: Destelbergen sludge; right: Deinze sludge; initial concentrations are given in g/l in legend) after overall parameter estimation.**

stress data using the objective function (8)

$$\sigma_e = \alpha \ln\left(\frac{C - C_C + \beta}{\beta}\right). \tag{12}$$

It was investigated whether one or more parameters of this logarithmic function could be constant for all experiments performed with the same sludge. Statistically, this was only possible for the parameter $\alpha$. The resulting functions together

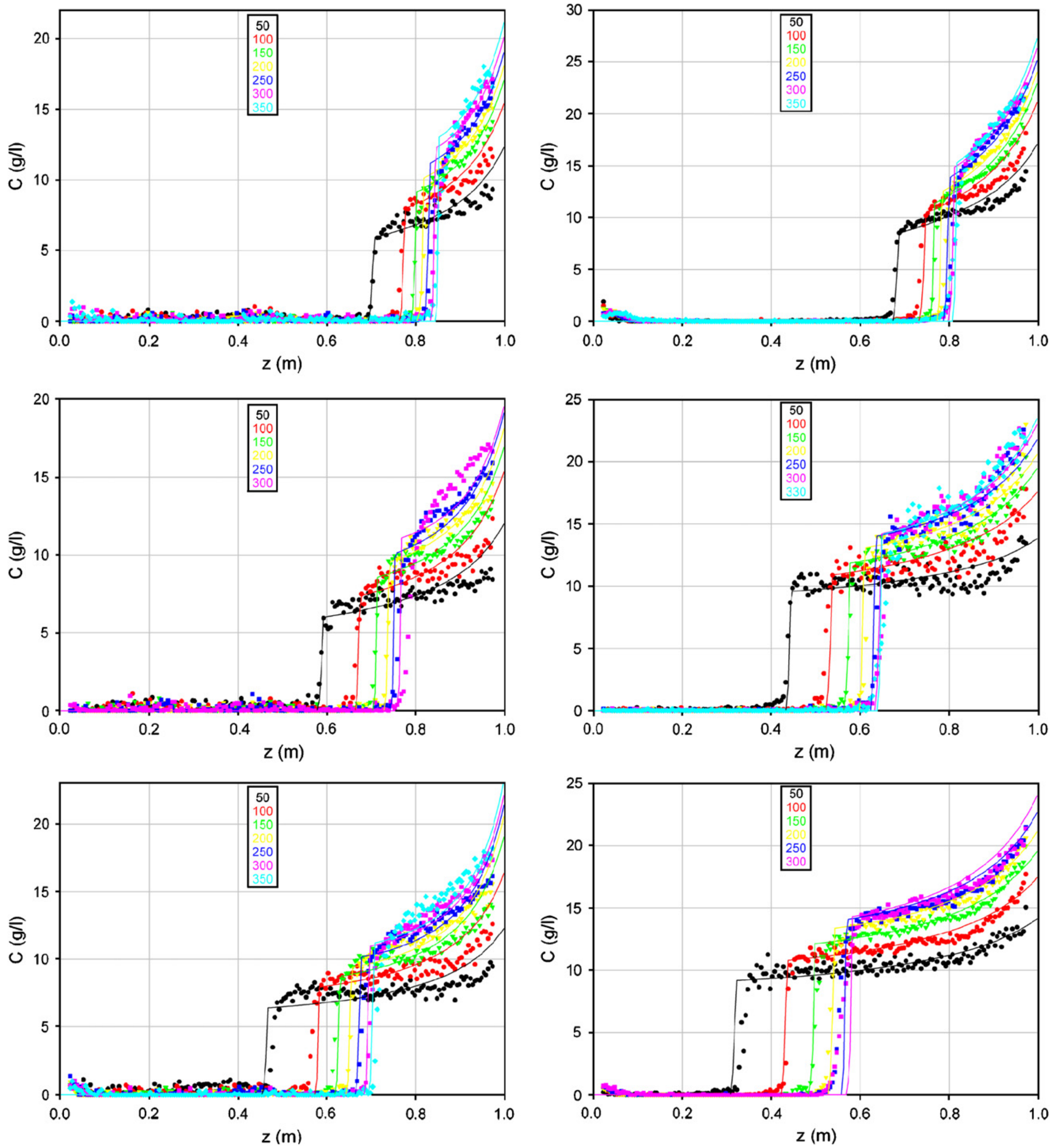


**Fig. 7 – Simulated (lines) and measured (symbols, De Clercq et al., 2005) solids concentration profiles in function of height at different times (indicated in minutes in legend) during batch settling of Destelbergen sludge (left; top: $C_0 = 2.40$ g/l; middle: $C_0 = 3.23$ g/l; bottom: $C_0 = 4.30$ g/l) and Deinze sludge (right; top: $C_0 = 3.67$ g/l; middle: $C_0 = 6.12$ g/l; bottom: $C_0 = 7.29$ g/l); simulations are performed with the batch settling model with the power function and the logarithmic function with the parameter values given in Table 2.**

with the calculated effective solids stress are presented in Fig. 5. The function (12) yields good results.

### 6.4. *Prediction/simulation of the batch settling experiments*

The inverse modelling showed that a power function for the Kynch batch density function combined with a logarithmic function for the effective solids stress could be used to describe the batch settling experiments. However, simulation of the measured solids concentration profiles with the obtained models and the parameter values obtained from the independent parameter estimations showed unsatisfying results (see De Clercq, 2006). Hence, an overall parameter estimation was performed using the obtained parameter values of both functions as initial guesses. The objective function for parameter estimation was the sum of squared errors (8) between the observed and predicted concentration profiles. Note that for one sludge, this resulted in a total of 250 000 data points that are used for the parameter estimation. The optimal parameter estimates are given in Table 2.

Measured and simulated batch settling curves are shown in Fig. 6 and solids concentration profiles at different time instants during the batch settling experiment in Fig. 7. The batch settling model describes the solids concentration profiles and the batch settling curves very well (sum of squared errors decreased with at least a factor 3 in comparison with the initial parameter guesses). The batch settling model characterized by the Kynch batch density function, the effective solids stress function and the compression solids concentration evolution is shown in Fig. 8 for both sludges.

The excellent description of the solids concentration profiles indicates a good potential for wider applicability of the model:

- By performing batch settling experiments at lower solids concentrations with a settling velocity lower than 250 m/d (i.e. located in the increasing part of the settling flux function), a Kynch batch density function can be found which also describes the settling (and the concentration profiles) at these lower solids concentrations.
- In practice, extensive experimental data as collected in De Clercq et al. (2005) are not available to identify the settling

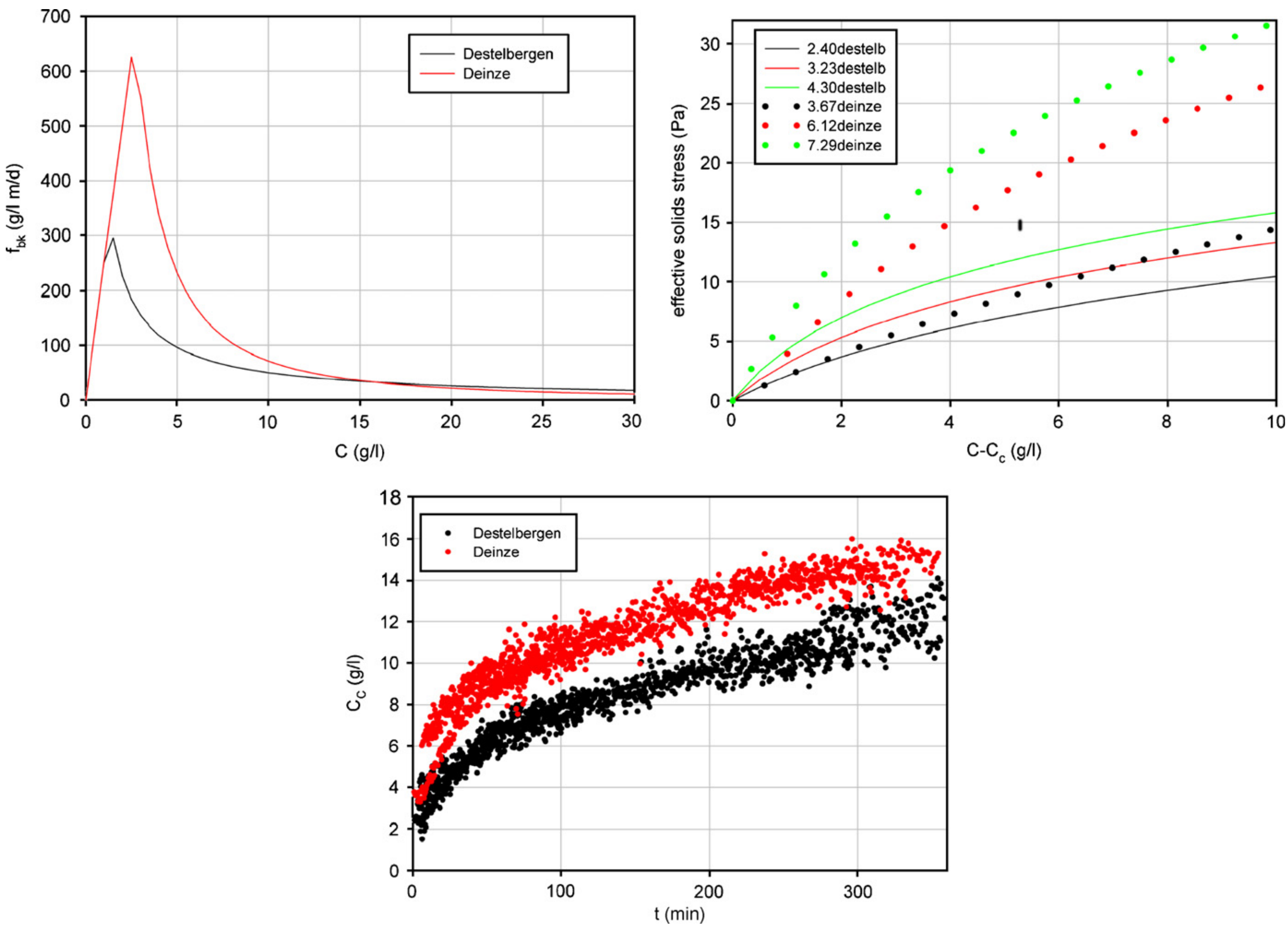


**Fig. 8 – Kynch batch density function (top left), effective solids stress function (top right; different concentrations are shown in legend in g/l) and evolution of the compression solids concentration (bottom) for batch settling of Destelbergen and Deinze sludge.**

behaviour, but batch settling curves with different initial solids concentrations are. At least three such curves need to be measured, i.e. at three quite different solids concentrations, in order to estimate the parameters of the model.

- When the settling behaviour, i.e. the parameters of the batch settling model, is identified, this settling model can be used to simulate continuous settling (1D, 2D or 3D). The time-dependent compression solids concentration can be located around the sludge blanket height in continuous settling, as shown in De Clercq (2006).

## 7. Conclusion

The batch settling experiments of De Clercq et al. (2005) were used to find appropriate functions for the Kynch batch density function and the effective solids stress. Inverse modelling and calculated initial settling velocities showed that a power function for the Kynch batch density function gives a single effective solids stress function throughout the experiments with a time-dependent compression solids concentration $C_C$. This time-dependency was already observed in the batch settling experiments, as well as the fact that the compression solids concentration was located close to the sludge blanket height. Inspection of the batch settling experiments allowed easy determination of the compression solids concentration evolution. Given the Kynch batch density function and the time-evolution of the compression solids concentration $C_C$, the functional relationship of the effective solids stress could be determined from inverse modelling. The effective solids stress was shown to exhibit a logarithmic behaviour with the solids concentration. The parameters of both the Kynch batch density function and the effective solids stress function were subsequently estimated from the solids concentration profiles.

The model describes the settling behaviour significantly better than any other model reported in literature and this for sludges originating from two different wastewater treatment plants.

## REFERENCES


Been, K., Sills, G.C., 1981. Self-weight consolidation of soft soils: an experimental and theoretical study. Géotechnique 31, 519–535.

Bergström, L., 1992. Sedimentation of flocculated alumina suspensions: $\gamma$-ray measurements and comparison with model predictions. J. Chem. Soc. Farad. Trans. 88 (21), 3201–3211.

Bürger, R., Karlsen, K.H., 2001. On some upwind difference schemes for the phenomenological sedimentation-consolidation model. J. Eng. Math. 41, 145–166.

Bürger, R., Concha, F., Tiller, F.M., 2000a. Applications of the phenomenological theory to several published experimental cases of sedimentation processes. Chem. Eng. J. 80, 105–117.

Bürger, R., Evje, S., Karlsen, K.H., Lie, K.-A., 2000b. Numerical methods for the simulations of the settling of flocculated suspensions. Chem. Eng. J. 80, 91–104.

Bürger, R., Concha, F., Fjelde, K.-K., Karlsen, K.H., 2001. Numerical simulation of the settling of polydisperse suspensions of spheres. Powder Technol. 113, 30–54.

Bürger, R., Damasceno, J.J.R., Karlsen, K.H., 2004. A mathematical model for batch and continuous thickening of flocculent suspensions in vessels with varying cross section. Int. J. Mineral Process. 73, 183–208.

Bustos, M.C., Concha, F., Bürger, R., Tory, E.M., 1999. Sedimentation and Thickening, Phenomenological Foundation and Mathematical Theory. Mathematical Modelling: Theory and Applications, vol. 8. Kluwer Academic Publishers, Dordrecht.

Cho, S.H., Colin, F., Sardin, M., Prost, C., 1993. Settling velocity of activated sludge. Water Res. 27, 1237–1242.

Cole, R.F., 1968. Experimental evaluation of the Kynch theory. Ph.D. Thesis, University of North Carolina, Chapel Hill.

Concha, F., Bürger, R., 2002. A century of research in sedimentation and thickening. KONA Powder and Particle 20, 38–70.

De Clercq, J., 2006. Batch and continuous settling of activated sludge: in-depth monitoring and 1D compression modelling. Ph.D. Thesis, Department of Chemical Engineering, Ghent University, ISBN 90-8578-093-4. Download: ⟨http://biomath.ugent.be/publications/download/declercqjeriffa_phd.pdf⟩.

De Clercq, J., Jacobs, J., Kinnear, D.J., Nopens, I., Dierckx, R.A., Defrancq, J., Vanrolleghem, P.A., 2005. Detailed spatio-temporal solids concentration profiling during batch settling of activated sludge using a radiotracer. Water Res. 39 (10), 2125–2135.

Diplas, P., Papanicolaou, A.N., 1997. Batch analysis of slurries in zone settling regime. J. Environ. Eng. 123 (7), 659–667.

Dochain, D., Vanrolleghem, P.A., 2001. Dynamical Modelling and Estimation in Wastewater Treatment Processes. IWA Publishing, London.

Dupont, R., Henze, M., 1992. Modelling of the secondary clarifier combined with the activated sludge model no. 1. Water Sci. Technol. 25 (6), 285–300.

Engquist, B., Osher, S., 1981. One-sided difference approximations for nonlinear conservation laws. Math. Comput. 36 (154), 321–352.

Evje, S., Karlsen, K.H., 2000. Monotone difference approximations of BV solutions to degenerate convection-diffusion equations. SIAM J. Numer. Anal. 37 (6), 1838–1860.

Font, R., 1991. Analysis of the batch sedimentation test. Chem. Eng. Sci. 46 (10), 2473–2482.

Font, R., Laveda, M.L., 2000. Semi-batch test of sedimentation. Application to design. Chem. Eng. J. 80, 157–165.

Grijspeerdt, K., Vanrolleghem, P., Verstraete, W., 1995. Selection of one-dimensional sedimentation models for on-line use. Water Sci. Technol. 31 (2), 193–204.

Härtel, L., Pöpel, H.J., 1992. A dynamic secondary clarifier model including processes of sludge thickening. Water Sci. Technol. 25 (6), 267–284.

Holdich, R.G., Butt, G., 1997. Experimental and numerical analysis of a sedimentation forming compressible compacts. Separation Sci. Technol. 32 (13), 2149–2171.

Jin, B., Wilén, B.-M., Lant, P., 2003. A comprehensive insight into floc characteristics and their impact on compressibility and settleability. Chem. Eng. J. 95 (1–3), 221–234.

Karl, J.R., Wells, S.A., 1999. Numerical model of sedimentation/thickening with inertial effects. J. Environ. Eng. 125 (9), 792–806.

Kinnear, D.J., 2002. Biological solids sedimentation: a model incorporating fundamental settling parameters. Ph.D. Thesis, Department of Civil and Environmental Engineering, University of Utah.

Kynch, G.J., 1952. A theory of sedimentation. Trans. Farad. Soc. 48, 166–176.

LeVeque, R.J., 1992. Numerical Methods for Conservation Laws, second ed. Birkhäuser Verlag, Basel, Switzerland, ISBN 3-7643-2723-5.

Marquardt, D.W., 1963. An algorithm for the least squares estimation of non-linear parameters. J. SIAM Control 11, 431–441.

Mines, R.O., Vilagos, J.L., Echelberger, W.F., Murphy, R.J., 2001. Conventional and AWT mixed-liquor settling characteristics. J. Environ. Eng. 127 (3), 249–258.

Otterpohl, R., Freund, M., 1992. Dynamic models for clarifiers of activated sludge plants with dry and wet weather flows. Water Sci. Technol. 26 (6), 1391–1400.

Shih, Y.T., Gidaspow, D., Wasan, D.T., 1986. Sedimentation of fine particles in nonaqueous media: Part I—Experimental, Part II—Modeling. Colloids and Surfaces 21, 393–429.

Shirato, M., Kato, H., Kobayashi, K., Sakazaki, H., 1970. Analysis of settling of thick slurries due to consolidation. J. Chem. Eng. Japan 3 (1), 98–104.

Takács, I., Patry, G.G., Nolasco, D., 1991. A dynamic model of the clarification-thickening process. Water Res. 25 (10), 1263–1271.

Tiller, F.M., Hsyung, N.B., Shen, Y.L., 1991. Catscan analysis of sedimentation in constant pressure filtration. Proceedings of the Fifth World Congress on Filtration 2, 80, Société Française de Filtration, Nice, France.

Vesilind, P.A., 1968. Discussion of 'Evaluation of activated sludge thickening theories' by Dick, R.I., Ewing, B.B. J. Sanitary Eng. Div. ASCE 94 (SA1), 185–191.

Zheng, Y., Bagley, D.M., 1998. Dynamic model for zone settling and compression gravity thickeners. J. Environ. Eng. 124 (10), 953–958.

Zheng, Y., Bagley, D.M., 1999. Numerical simulation of batch settling process. J. Environ. Eng. 125 (11), 1007–1013.